Salience, Systemic Risk and Spectral Risk Measures as Capital Requirements*

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Abstract

In this paper, we evaluate the effectiveness of macroprudential capital requirements in the form of Value at Risk and three alternative spectral risk measures from the systemic risk perspective. Overall, we find that prudential instruments based on salience and overweighting of tail market losses are beneficial for policymakers aimed to limit the procyclicality in the financial sector and reduce the likelihood of the systemic crises. In the steady state, the financial sector exhibits risk-seeking attitudes when the risky asset upside is salient and risk-averse behavior when the downside is salient. In contrast, overweighting of almost certain market losses results in a rapid leverage acceleration and risk-seeking preferences, and exacerbates systemic risk. More important, focusing on both upside and downside risks fulfills the prudential objective of building a more resilient financial system. Our model illuminates how adverse liquidity and uncertainty shocks elicit policy responses, but also how they affect risk attitudes and the time and cross-sectional dimension of systemic risk.

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1 Introduction

The great recession and the market downturn of 2008 had various causes: overoptimism and asset price bubble, households indebtedness, securitization, and extensive use of complex derivatives and monetary excesses brought by low interest rates and financial markets globalization, to name a few (Mian and Sufi (2015); Stiglitz (2010)). However, there is an additional fundamental less-noticeable cause; our limited knowledge of how prices move and how risks evolve and materialize. The recent crisis illustrates that widespread failures and losses of financial institutions can adversely affect output and market stability and emphasized the importance of proper regulatory risk management. In a nutshell, a better evaluation of market risk and a more profound understanding of macroprudential regulation-systemic risk feedback and its spillovers is the goal of the paper.

The salient revelation of systemic risk defined as the system-wide cost of financial instability has received increasing attention in the post-crisis aftermath (Acharya (2009); Tirole (2011)). Prevalent measures of these costs include expected capital losses and the likelihood of systemic crisis (Acharya et al. (2012); Roukny et al. (2018)), fire sale externalities due to common asset exposure (Brunnermeier (2009); Caballero and Simsek (2013)), default cascades due to credit interlinkages (Eisenberg and Noe (2001)) and spillovers to the real economy.[1] The main objective of the macroprudential policy is to prevent financial crises by limiting the accumulation of vulnerabilities in the financial sector that give rise to systemic risk, but also to alleviate the consequences of systemic risk once it materializes. Importantly, financial crises can create a credit crunch for nonfinancial borrowers and lead to a decline in aggregate output (Laeven and Valencia (2010)).

In this paper, we aim to fill the important gap in the design of regulatory capital requirements by providing a conceptual framework and how they could be structured to account for systemic risk. We do so in two ways. First, we develop a heterogeneous agent equilibrium model with binding capital requirement constraint and an endogenous systemic risk in terms of probability of financial sector being undercapitalized. The model is a simplified version of Brunnermeier and Sannikov (2014) with capital requirement constraint. Implicit reason for regulation is that financial sector undercapitalization leads to fire sales and costly ex-post bailouts.[2] Second, we utilize the alternative models of risk attitudes in behavioral finance and economics, which have gained extensive recent empirical evidence in O'Donoghue and Somerville (2018) and relating it to regulatory risk measurement. Regulators implement capital requirements as macroprudential VaR or three alternative spectral risk measures. In a nutshell, we match the main implication that tail losses receive disproportionate weight in the regulatory risk measurement with the observation that probability weighting influences financial sector risk aversion or risk seeking. Doing so allows disentangling whether it is the financial sector's risk attitudes or regulatory risk measurement that affects the financial and economic sphere. We first analyze the steady state equilibrium. Then, we investigate the ex-post role of the macroprudential

[1] Bisias et al. (2012) provide a comprehensive survey of quantitative approaches for measuring systemic risk.
[2] Shleifer and Vishny (2011) explain the mechanisms through which fire sales exacerbate financial system stability.

instrument in the crisis management following a sudden increase in borrowing costs, a credit crunch and an uncertainty shock.

Probability weighting and salience introduce alternative sources of risk attitudes to the marginal utility of wealth from the expected utility framework. The key idea is that investors overweight or underweight outcomes relative to their objective probabilities.[3] For example, in prospect theory of Tversky and Kahneman (1992) investors overweight rare events and underweight almost certain events. The prospect theory captures the observation that people have "limited ability to comprehend and evaluate extreme probabilities". In salience theory, the intuition is that the payoffs that draw the decision maker's attention are "salient". The decision maker overweights salient payoffs, where his attention is drawn to payoffs which are most different relative to the average market payoff or the safe asset payoff (Bordalo et al. (2013b)). Probability weighting leads to overweighting tail (small probability) events and underweighting intermediate outcomes, while salience-based probability weighting can also lead to overweighting of intermediate outcomes depending on what attracts attention. These features generate more nuanced predictions compared to the expected utility, investors may exhibit risk-seeking and risk-averse behavior or changes in risk aversion. For instance, in the salience model of Bordalo et al. (2013a), outcomes are weighted according to how much they differ from the average in a given state of the world, with more extreme payoff states garnering more attention. As a consequence, the decision maker is risk-seeking when a lottery's upside is salient and risk-averse when its downside is salient. Empirically, in Cohn et al. (2015), authors rendered salient market boom or bust scenario in a controlled experimental environment and document countercyclical risk aversion where participants exhibit lower willingness to take risks during financial booms compared to busts.[4]

As previously mentioned, we consider three spectral risk measures as regulatory capital requirements. The prominent feature of spectral risk measures is that they relate the market risk measure to the investor's subjective risk spectrum. Importantly, the risk spectrum is an alternative representation of the probability weighting function or decision weights in Tversky and Kahneman (1992) and Bordalo et al. (2013b). Among viable applications, the literature has suggested using them to set capital requirements and to devise optimal portfolioAcerbi (2002), or for setting margin requirements (Cotter and Dowd (2006))[5]. First, we use Wang (2000)'s distortion function, which transforms the probability distribution by foremost overweighting tail probabilities and has been used in the literature to determine insurance premiums. Second, we construct decision weights in the spirit of Tversky and Kahneman (1992), where the decision

[3] In expected utility theory, the utility of each outcome is weighted by the probability of that outcome occurring. The basic idea of probability weighting and dual theory of choice under risk developed by Yaari (1987) is that individuals might use decision weights which differ from objective probabilities, and such deviations represent probability distortions.

[4] By their definition, countercyclical risk aversion is a lower willingness to buy risky assets while controlling for financial wealth, asset price, objective and subjective expectation about asset returns and volatility.

[5] In principle, spectral risk measures can be applied to problems involving decision making under risk. Practically relevant, spectral risk measures are a promising generalization of Expected Shortfall as a market risk measure (on Banking Supervision (2011)). This aspect of macroprudential regulatory reform, new market risk measures are yet to be introduced and implemented.

makers overweight market outcomes of small probabilities and underweight almost certain events. Third, we consider the opposite case where regulators underweight small probabilities and overweight almost certain events instead. We call these KT (Tversky and Kahneman (1992)) and anti-KT market risk measures, respectively. In principle, what constitutes the most salient state for regulators changes across different risk measures.

The focus of the literature and regulators so far has been on early warning indicators of aggregate financial imbalances, notably property and equity prices and credit-to-GDP (Drehmann et al. (2010))[6] and stabilization benefits of macroprudential policy instruments that respond to indicators to reduce this procyclicality, for instance countercyclical provisioning, loan-to-value ratio and capital requirements (Drehmann et al. (2010); Lambertini et al. (2013))[7]. Current regulatory framework Basel III has adopted countercyclical capital requirements with the credit-to-GDP ratio as an early warning indicator (Committee et al. (2010)). These are instruments designed to ensure the solvency of a financial institution by imposing a restriction to hold enough capital to cover expected future losses. In principle, banks accumulate a capital buffer in periods of high economic growth that can be employed in distress periods to absorb financial shocks and losses. Overall, countercyclical capital buffers can reduce the magnitude of a credit boom and alleviate a credit crunch (Drehmann et al. (2010)).

While investors may focus on downside risks as well as on upside risks, the purpose of current regulatory risk management is to quantify the downside risk potential associated with the loss of a certain portfolio. Capital requirements in a form of a risk measure are designed to compute required absorbency buffer for financial institutions to be able to absorb future market losses and insure solvency of these institutions. The pre-crisis measure of both market and systemic risk, Value at Risk yields a simple and elegant solution: it asks what is the worst-case scenario associated with portfolio outcome for a fixed confidence level. In other words, what is the maximum probable future loss for a given confidence level and a time horizon. In this respect, VaR capital requirements provide only partial insurance against insolvency as it neglects losses which can be expected when tail events occur[8]. The post-crisis measure of risk, Expected Shortfall measures the average losses beyond the VaR threshold and crucially both are quantile-based risk measures and special cases of spectral risk measures (Acerbi (2002)). More precisely, the spectral risk measure is a weighted average of the quantiles of a loss distribution, where the corresponding weighting function is the investor's risk aversion. In this respect, VaR overweights a singular loss at the fixed confidence level and underweights the residual losses. Similarly, Expected Shortfall overweights all losses beyond VaR threshold and

[6]Shin (2013) suggest that the financial sector procyclicality provides a framework for selecting indicators of vulnerability to crises, and thus compares predictive power of market prices indicators (CDS spreads and implied volatility), credit to GDP and banking sector liabilities aggregates. More broadly, Chamon and Crowe (2012) provide a recent comprehensive summary of early warning signals to predict general crises covering external, financial, real, and fiscal variables, as well as institutional factors and various measures of contagion.

[7]For instance, Lambertini et al. (2013) report loan-to-value ratio that responds countercyclically to credit growth smooths credit supply cycle, inflation and output growth. More broadly, Elliott (2011) provides a detailed discussion of available macroprudential instruments.

[8]One concern in the recent crisis has been the failure of VaR to measure potential tail losses in the mortgage-backed securities (Acharya et al. (2012)). This feature is relevant when considering the moral hazard of banks since realized tail losses beyond VaR threshold are compensated by the costly government capital injection.

underweights the remaining ones.

However, fewer works are devoted to understanding how macroprudential capital requirements in the form of a risk measure instead affect the systemic risk, economic activity, credit and asset prices endogenously. In its origin, the intrinsic source of systemic risk is the procyclicality of the financial system, in that endogenous credit market conditions amplify and propagate shocks to the real sector.[9] Although credit availability and external finance premium may amplify business cycle fluctuations, financial accelerator mechanism may fail to generate widespread financial distress with subsequent unprecedented economic contraction. As suggested erstwhile by Borio et al. (2001), an additional source of procyclicality may be the changes in the risk measurement and risk perception of the financial sector over time. The pattern of procyclical risk perception in that investors are overly risk-averse during financial downturns has been empirically documented as commercial and investment banks re-balanced portfolio towards risk-free assets (Borio (2014)). In addition, a slow increase in regulatory risk-weighted assets comparing to a rapid increase in total assets during credit booms is indicative of the procyclical regulatory risk measurement and perception (Adrian and Shin (2010)). Overall, these suggest that measured and perceived risk in the financial system decline in business cycle expansions and increases in recessions. Procyclical risk assessment leads to risk being underestimated in booms and financial institutions holding insufficient capital to cover market losses. In summary, understanding systematic changes in financial sector risk perception and regulatory risk measurement seems to be the key for macroprudential policy design aimed to reduce the risk of financial instability and associated macroeconomic costs that arise from the financial sector procyclicality.

Accounting for endogenous feedback between asset prices and capital requirements is necessary since by marked-to-market accounting changes in asset prices immediately manifest as balance sheet gains or losses (Adrian and Shin (2010)) and asset prices being an important indicator of the stage of the financial cycle (Borio (2014)). Moreover, systemic risk - the risk of a widespread failure of financial institutions - rarely arises from the contagion effects owing to purely individual institution-specific factors. More often, systemic risk has the roots in banks underestimating their exposure the aggregate financial and business cycle in the economy (Borio et al. (2001)) and entails higher output loss.[10] Inadequate risk measures might defacto exacerbate systemic risk, in contrast to systemic risk being considered the primary reason for the supervision and regulation of the banking industry in the first place.

We find that macroprudential VaR based on overweighting of small probabilities produces the divergence in risk attitudes of two sectors towards the downside market risk and downside balance sheet risk. Perhaps at the most basic level, the financial sector exhibit both risk-loving and risk-averse behavior, depending on the context. When the risky asset upside is salient, the financial sector is risk-seeking, and conversely risk-averse when its downside is salient. On the other hand, the nonfinancial sector is always risk-averse. At a broader level, two sectors per-

[9] The empirical evidence implies procyclicality; high economic growth correlates with substantial increases in credit and equity and property prices and with a decrease in corporate bond spreads (Borio (2014)).

[10] For instance, Laeven and Valencia (2010) estimate a 25 percent output loss following the great recession.

ceive market risks differently even though they possess the same information. In this respect, our result is comparable to "local thinkers" in Gennaioli and Shleifer (2010), Bordalo et al. (2013b) and Tversky and Kahneman (1992), where decision-makers overweight the information they focus on and do not consider all the available information. Such decision-makers are known as local thinkers because they neglect potentially important but unrepresentative data. Similarly, the financial sector overweights states that draw their attention and neglect states that do not. Unlike financial friction models in the spirit of Bernanke and Gertler (1990) which are used to study financial imbalances and systemic risk, we emphasize the heterogeneity in information *processing* instead of heterogeneity in information *availability*.

Overall, we find that the implementation of the macroprudential policy needs to be fitted to the measure of systemic risk. For instance, we find support for the ex-ante preventive role of prudential instruments based on overweighting of tail market losses. Both *VaR* and Wang's risk measure are beneficial for policymakers aimed to limit the procyclicality in the financial sector and reduce the likelihood of the systemic crises. In contrast, focusing at the same time on upside and downside market risks with anti-KT and KT results in a rapid leverage acceleration and exacerbates systemic risk. On the other hand, fulfills the prudential goal of building a more resilient financial system in crisis resolution. The results suggest that *VaR* and Wang's risk measure are effective in controlling endogenous risk and the likelihood of systemic crisis in environments prone to uncertainty shocks, but unable to prevent output decline and fire sales. By comparison, with KT risk measure output increases in the face of uncertainty shock. Under VaR and Wang's regulation fire sales are mitigated in environments prone to liquidity shocks. These policies cannot eliminate fire sales, but they can mute their effects. In contrast Brunnermeier and Sannikov (2014) where the financial sector is unregulated, fire sales do not lead to an increase in financial system instability. In managing adverse inside or outside liquidity shocks, all four measures can either reduce the ex-post crisis anticipation or amplification, but cannot control both.

The overall conclusion we derive is that the implementation of macroprudential capital requirements needs to be fitted to limit the likelihood and the costs of systemic crises triggered by adverse shocks. Capital buffers that adequately weigh ex-ante prevention of systemic risk and ex-post crisis management might be capable of achieving this mission. In our framework, this objective translates into weighing the downside risk measures and upside risk measures according to regulators' preferences for risk reduction or resilience. Alternatively, regulators may enforce ex-ante prudential framework with *VaR* or Wang's risk measure during peaceful times while adjusting the choice of a risk measure during crises management, where for instance, mitigating systemic risk takes precedence over output loss or vice versa.

Understanding how different adverse shocks elicit policy response and information processioning and result in various measures of systemic risk is one of the main contributions of this paper. For instance, systemic risk in terms of likelihood of the manifests when the inside liquidity of the financial sector is impaired and regulators respond to liquidity shortage by decreasing capital requirements. The endogenous risk seems to have its roots in experts overweighting their exposure to systemic events after a negative outside liquidity shock. Fire sales emerge when an economy is hit by a sudden uncertainty shock and inevitably lead to a recession with low output growth. One possible reason for this context-dependent policy effectiveness is the context-dependent focus of agents, as they overweight states that draw their attention.

The observation that different market risk measures may elicit different risk attitudes can potentially have surprising implications on the role of regulatory risk measurement in limiting systemic risk. We argue that regulators may benefit from implementing two-pillars policy. While the design of regulatory tools has been primarily focused on the level of required capital requirements, regulators should possibly adopt an additional role of anchoring expectations about future market losses. This goal may be achieved by exploiting a cross-sectional and time dimension of market risk. As different sectors assign different decision weights in comparison to regulators, they distort current or expected future market losses. The prudential authority may quantify short-term market risk and communicate expectations of expected future risk to set the term structure of market tail risk. This view resembles the expectation channel through which monetary policy anchor expectations of the long-term interest rate by setting the current interest rate and communicating future expected short-term rates. [11] In this way, anchoring of decision weights across sectors and through time may be a promising direction for future prudential policy design.

In the following, Section 2 describes risk measures as a key method to quantify expected losses. Section 3 and Section 4 describe the equilibrium model with a macroprudential VaR and analyze the effectiveness of this policy from the systemic risk perspective after liquidity and uncertainty shocks. We also investigate the benefits and weaknesses of tighter regulation. Section 5 presents an alternative regulation in the form of three spectral risk measures and studies their advantages and disadvantages. Section 6 concludes.

2 Risk measures

In this section, we briefly define spectral risk measures which will be used in section 3 and 5 to measure market risk and devise regulatory capital requirements. The key idea of alternative regulation is probability overweighting, where regulators overweight losses which are salient to them.

In a world of a continuous marked to market accounting of the financial intermediary balance sheet, price changes instantaneously manifest as market gains and losses. Figure 1 depicts market realized returns and realized historical volatility preceding, during, and following the market downturn of September-December 2008. Positive and negative returns reflect market gains (upside risk) and losses (downside risk), while an increase in volatility affects both risks symmetrically. During the crisis, the probability distribution of market gains and losses

[11] The underlying mechanism through which anchoring is achieved is the expectation hypothesis. This hypothesis postulates that the long-term rate depends solely on current and future expected short-term rates, such that the expected final wealth after continually reinvesting in short-term bonds equals the final wealth when investing in long-term bonds.

widened and volatility spiked far above the pre-crisis average(Figure 2). To the extent that both upside and downside risk may fluctuate, traditional risks measures such as variance popularized by Markowitz (1952) may not be sufficient in the context of capital requirements. Spectral risk measures are flexible enough to include not just the second moment of the probability distribution(variance) but also higher and lower order moments (mean), depending on the risk spectrum.

Spectral risk measures are defined as the weighted average of quantiles of a loss probability

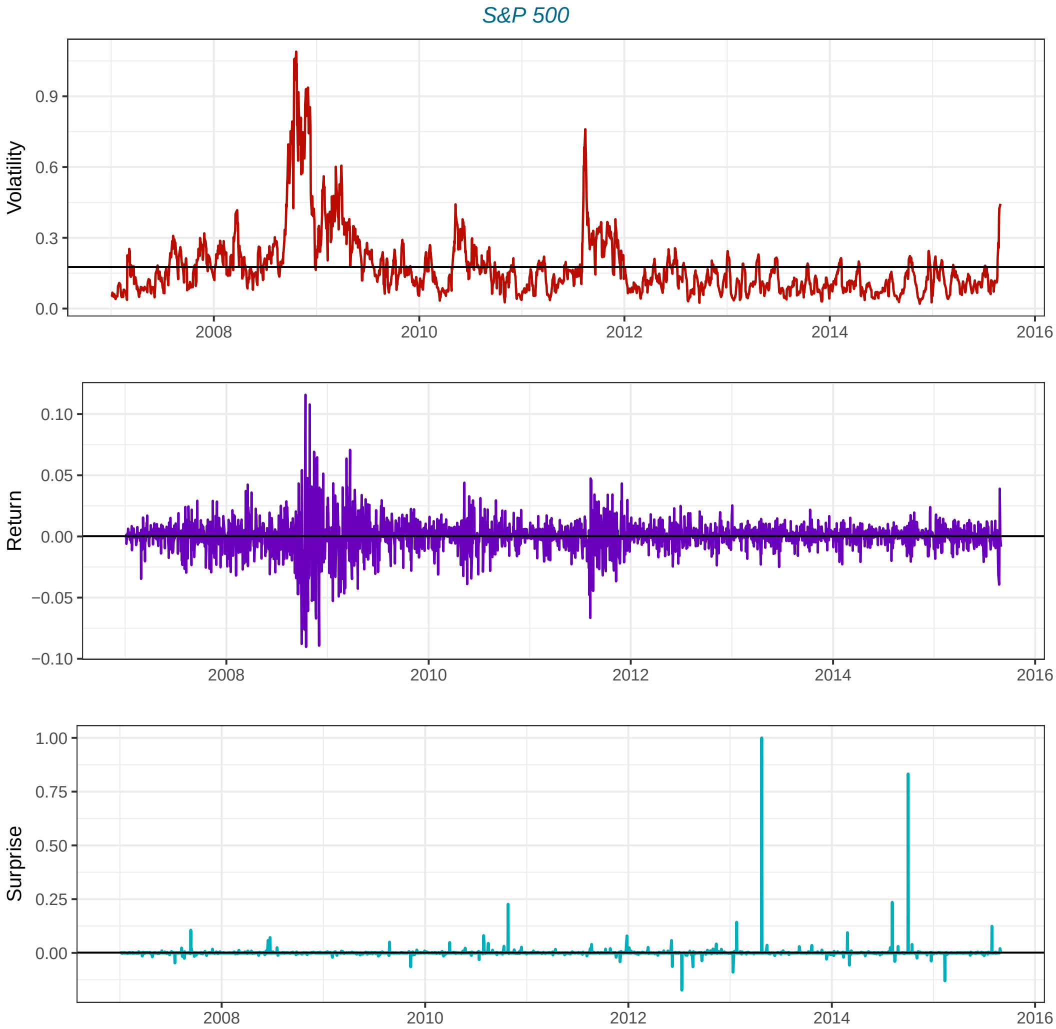


Figure 1: Realized market return and volatility

distribution

$$M_g(X) = \int_0^1 g(p)F^{-1}(p)dp \tag{1}$$

where $F^{-1}(p)$ is a quantile function of a random variable X which measures market losses[12] , and $g(p)$ satisfies

[12] Quantile at level p is a an inverse of cumulative distribution function of a random variable X, that is $F^{-1}(p) = \inf\{x \quad : F(x) \equiv Prob\,[X \leq x] \geq p\}$.

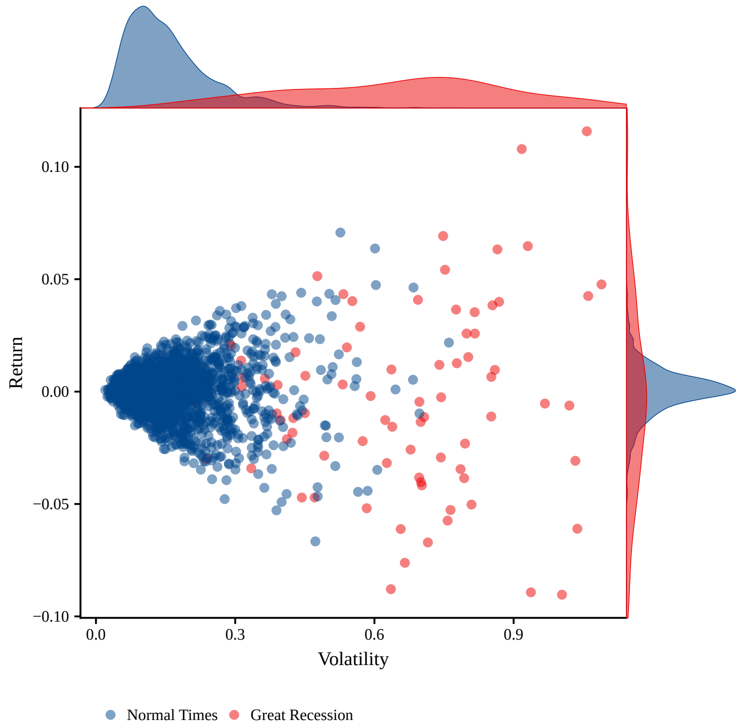


Figure 2: Probability distributions

1. $g(p) \geq 0 \quad$ (positivity),
2. $\int_0^1 g(p)dp = 1 \quad$ (normalization), and
3. $g'(p) \geq 0 \quad$ (monotonicity).

The weighting function $g(p)$ is called the risk spectrum and reflects the regulatory degree of risk aversion or seeking. It is related to the probability weighting function or decision weights in Tversky and Kahneman (1992) such that $G'(p) = g(p)$ holds. The first condition requires that the weights are weakly positive, the second assumes that the sum of the weights is one[13], but the key condition is the third one. This is a direct reflection of risk aversion and requires that the weights attached to larger losses are no less than the weights attached to smaller losses. In this respect, we would expect the weights to rise smoothly. The higher is the loss aversion of regulators, the faster the weights rise.[14]

In salience theory, the individual's attention is drawn to the most surprising or different market outcomes which he overweights in his decisions. When quantifying market risk with VaR_α, a loss which can occur with a small probability is salient for regulators. VaR_α fails to

[13]This property reflects diversification benefits, sub-additivity implies when combining assets with downside risk into a portfolio, total portfolio loss is lower or equal to the sum of individual losses. Spectral risk measures satisfy all four properties of a coherent risk measure, where sub-additivity is satisfied with equality. For the precise definition of a risk measure and four properties, see Appendix A.

[14]However, there still remains the question of how to specify the weighting function, and one proposed way in the literature is from the agent's utility function.

satisfy monotonicity condition as it overweights the loss at fixed confidence level α and underweights higher losses.[15] As it will be explained in what follows, VaR_α risk measure by its definition is inversely proportional to the ratio of *unexpected* and *expected* market losses. In this respect, we can interpret VaR_α as how "far into tails" realized market losses are, in other words, how surprising they are. In essence, by setting the fixed confidence level regulators aim to control the probability of being surprised but not the level of surprise of the market losses. As we will see in section 5, only Wang (2000)'s risk measure out of four measures we consider satisfies this property. Both KT and anti-KT violate monotonicity condition.

In the following section, we use the definition of spectral risk measures to obtain VaR capital requirements in heterogeneous agent model.

3 Model

The model is a simplified version of Brunnermeier and Sannikov (2014), with the capital requirement constraint imposed on the financial sector and capital as a single factor of production. There are two types of agents, unconstrained risk-averse households and constrained risk-averse experts. We depart from representative agent assumption and introduce heterogeneity in productivity and impatience together with aggregate risk, minimum assumptions needed to obtain fire sales, systemic risk and borrowing in equilibrium. The constraint limits the amount of borrowing depending on asset side balance sheet risk, measured as a macroprudential VaR_α or as a spectral risk measure. We first derive the steady state equilibrium with optimal consumption and investing choices of two agents and endogenous systemic risk. We summarize equilibrium equations in subsection 3.3 and study equilibrium dynamics in section 4. Then, in subsection 4.1,4.2 and 4.3, we study new steady state following unexpected, permanent shocks such as an increase in borrowing costs, a credit crunch, or uncertainty shock. We also study tradeoffs which arise after adopting a tighter or more accommodative VaR policy in subsection 4.4.

There is a continuum of infinitely-lived households and experts with preferences represented by the utility function

$$E\left[\int_0^\infty e^{-rt}\log \underline{c}_t dt\right], \tag{2}$$

$$E\left[\int_0^\infty e^{-\rho t}\log c_t dt\right]. \tag{3}$$

where $\underline{c}_t$ and c_t are households' and expert consumption in the current period. Both agents produce final good from the capital in a linear fashion

$$y_t = ak_t, \tag{4}$$

$$\underline{y}_t = \underline{a}\,\underline{k}_t, \tag{5}$$

[15] $g(\alpha) = 1$ for fixed α, and $g(p) = 0$ for $\alpha \neq p$.

with experts being more productive ($a > \underline{a}$) and impatient ($\rho > r$) than households. Capital supply is exogenous and evolves over time according to a geometric Brownian motion

$$\frac{dk_t}{k_t} = \sigma dW_t, \tag{6}$$

where W_t is a standard Brownian motion. The term σdW_t is called *capital quality* shock and captures temporary random changes in expectation about future productivity of capital.[16] Let p_t be the price of capital, which will be endogenous in equilibrium. In principle, experts can finance a capital purchase by issuing debt

$$b_t = p_t k_t - n_t,$$

where n_t denotes expert's net worth. At each time period, experts choose how much to consume and borrow, so experts net worth evolves as

$$dn_t = ak_t d_t + d(p_t k_t) - r_t(p_t k_t - n_t)dt - c_t dt. \tag{7}$$

The first two terms are income from production and capital gains or losses which reflect changes in the market value of the risky asset. The second two terms are debt repayment and consumption. Net worth is endogenous since it depends on the consumption and borrowing decision and the endogenous asset price evolution

$$\frac{dp_t}{p_t} = \mu_t^p dt + \sigma^p dW_t. \tag{8}$$

Using the Ito's product rule, market gains and losses evolve as

$$\frac{d(p_t k_t)}{p_t k_t} = (\mu_t^p + \sigma\sigma_t^p)dt + (\sigma + \sigma_t^p)dW_t. \tag{9}$$

3.1 Experts Optimization

In standard frictionless models in which Modigliani-Miller theorem holds, firms maximize the present discounted value of future profits or shareholders value.[17] Instead, experts maximize a lifetime utility of consumption, and moreover, the macroprudential regulation limits the level of external debt financing by forcing experts to hold enough net worth to absorb expected future losses.

Now let us consider how Value at Risk or a spectral risk measure enters the expert's optimization problem. Recall that both risk measures quantify the downside market risk potential.

[16]A diffusion process (a geometric Brownian motion) is a useful approach for modeling uncertainty or risk because of its tractability. In particular, due to Ito's lemma functions and products of diffusion processes are also diffusion processes and we can easily calculate their drift and volatility.

[17]Modigliani-Miller theorem states that capital structure(debt or equity financing) is irrelevant for the value of the firm if the firm maximizes shareholder value, there is no arbitrage, asymmetric information, and borrowing frictions.

We apply these measures to a loss process defined here as a market loss on the asset side of the balance sheet as follows. Let $X_t \equiv p_t k_t$ denote the market value of capital at time t so

$$dX_t = X_t(\mu_t^p + \sigma\sigma_t^p)dt + X_t(\sigma + \sigma_t^p)dW_t$$

follows a geometric Brownian motion. Solving for this stochastic differential equation we have

$$X_t = X_0 + \int_0^t X_s(\mu_s^p + \sigma\sigma_s^p)ds + \int_0^t X_s(\sigma + \sigma_s^p)dW_s,$$

$$X_t = X_0 \exp\left(\int_0^t (\mu_s^p + \sigma\sigma_s^p - \frac{1}{2}(\sigma + \sigma_s^p)^2)ds + \int_0^t (\sigma + \sigma_s^p)dW_s\right),$$

which further implies by the independence of increments property of a geometric Brownian motion

$$X_{t+\tau} = X_t \exp\left(\int_t^{t+\tau} (\mu_s^p + \sigma\sigma_s^p - \frac{1}{2}(\sigma + \sigma_s^p)^2)ds + \int_t^{t+\tau} (\sigma + \sigma_s^p)dW_s\right).$$

$X_{t+\tau}$ is the future market value of capital at time $t+\tau$ if the capital exposure between time t and $t+\tau$ were kept unchanged. We define balance sheet loss between the period t and $t+\tau$ as

$$Loss(t, t+\tau) \equiv X_t - X_{t+\tau}. \tag{10}$$

The interpretation is that banks assets are marked-to-market, and therefore marked-to-market gains and losses between two successive periods are captured by the change in the market value of capital between two periods $X_t - X_{t+\tau}$. Since market losses are stochastic, when evaluating downside risk VaR_α computes the maximum loss over the horizon τ which can be exceeded only with a small fixed probability α if the current portfolio were kept unchanged.

$$VaR_\alpha^{t,t+\tau} = \inf\{L \geq 0 : P(X_t - X_{t+\tau} \geq L|\mathcal{F}_t) \leq \alpha\}. \tag{11}$$

In other words, VaR_α is the $1-\alpha$ quantile of a market loss distribution

$$P(Loss(t, t+\tau) \leq VaR_\alpha^{t,t+\tau}) = 1 - \alpha.$$ [18]

Importantly, we make the risk computations consistent with practice when assessing the risk of a portfolio by assuming that the current portfolio composition is kept unchanged and current market conditions will prevail over the horizon τ. This means we *condition on information available at time t*, and project it to future periods when assessing future market risk. As a result, a risk measure such as backward-looking $VaR_\alpha^{t,t-\tau}$ instead, compromise different information set where *past* portfolio holdings and market conditions are relevant for future risk assessment and would imply a different regulatory framework.

[18] For instance, in case α= 0.05, $VaR_\alpha^{t,t+\tau}$ implies that there is 95% probability that the market loss will not exceed the VaR threshold and 5 % probability of experiencing a market loss larger than VaR.

Proposition 1. *We have*

$$VaR_{\alpha}^{t,t+\tau} = p_t k_t (1 - \mathrm{e}^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau + \Phi^{-1}(\alpha)(\sigma+\sigma_t^p)\sqrt{\tau})})$$

where $\Phi^{-1}(\cdot)$ is the inverse of the cumulative distribution function of the standard normal distribution.

PROOF. See the Appendix A. □

Similarly, a spectral risk measure is defined as a weighted average of VaR quantiles

$$M(t) = \int_0^1 g(p) VaR(p)^{t,t+\tau} dp$$

For different choices of a weighting function $g(p)$, we obtain different regulatory risk assessment. We assume that expert's borrowing is constrained by

$$p_t k_t M(t) \leq n_t. \tag{12}$$

In summary, experts choose k_t and c_t to maximize (3) subject to net worth evolution (7) and the regulatory capital requirement constraint (12). If we define leverage by $\frac{p_t k_t}{n_t}$, the constraint puts the bound on leverage experts can take depending on current market conditions, and therefore it is a state-varying borrowing constraint. As a result, the amount of external debt financing depends on evaluated regulatory future losses. If experts had an unconstrained choice of capital holdings, they would issue debt up to the point where leverage equals the Sharpe ratio of a productive risky asset.

It is therefore worthwhile to pause and devise more micro-founded rationale why this type of borrowing constraint is imposed on experts in the first place. In general, in principal-agent problems, limited pledgeability generates the borrowing constraint. Limited pledgeability is a consequence of information frictions which arise from asymmetric information between a borrower and lender such as adverse selection and moral hazard.[19] In an environment where future cash flows or return on investment are not observable or costly verifiable, lenders can pledge only part of future income. To finance investment, a borrower can use internal funds, issue riskless debt or outside equity. Influential pecking order theory in financing in Myers and Majluf (1984) implies that funds such as retained earnings are preferred over debt while debt is preferred over outside equity. The key idea which drives the result is information sensitivity of outside equity and insensitivity of debt and internal funds. In effect, information friction and capital structure interact.

Unlike the cases where limited pledgeability arises due to private information, limited pledgeability here occurs because the future price of capital can decline or become more volatile,

[19] Information frictions, in general, assume either ex-post or ex-ante information asymmetry between a borrower and a lender. With ex-post private information, a borrower knows about the value of the project after undertaking the investment, but a lender can infer the value by auditing at a physical cost (Gale and Hellwig (1985); Townsend (1979)). In the second case, managers know more about the value of a firm's existing assets or a new project outcome before undertaking the project (Myers and Majluf (1984)).

that is the value of collateral can change even if households and experts have the same public information. Because the information environment changes, the more surprising realized market losses are, the larger the required capital buffer. As a consequence, financing choice shifts from debt to internal funds. Closer inspection reveals we can perceive a risk measure as a measure of information sensitivity. For example, by definition macroprudential VaR_α is inversely proportional to the ratio of expected and unexpected market losses and can be related to the distorted signal-to-noise ratio in information theory literature.[20] It tells us that when we see abnormally high prices it is because of volatility (noise) component and not because of the trend (signal) component.[21] Therefore, we can interpret VaR as a measure of information sensitivity since it conveys the degree of .[22] By setting a fixed confidence level, regulators aim to control the probability of surprise but not the level of surprise of market losses. In essence, the capital requirement should suffice in providing a buffer to a distorted expectation of future market losses.

Another suggestive connection is between a risk measure and an efficient portfolio. By Markowitz (1952)'s definition, for each level of risk investor contemplate, it is possible to construct an efficient portfolio which yields the highest expected return. The reverse also holds, for each level of a return investor targets, there exists an efficient portfolio with the lowest risk. In effect, the expected return-risk tradeoff arises.[23] The particular portfolio choice depends on investor's risk attitudes, with it ranging from an aversion to seeking and corresponding to investing total wealth in a risk-free asset, in both risky and risk-free asset or leveraging to buy the risky asset. It is straightforward to notice a resemblance between the efficient portfolio and a risk measure. In particular, we may think of a "reward" for regulators as being equal to expected losses whereas a "risk" as being equal to unexpected losses. Regulators are risk-averse when in their assessment of the market risk they overweight unexpected losses and risk-seeking when they overweight expected losses. A risk measure entails tail loss aversion or seeking function $g(\cdot)$, and by specifying this function losses are distorted by different macroprudential regulation. As we will see in the following sections, the regulatory tail loss aversion affects the risk attitudes of the financial sector in terms of the marginal utility of wealth.

3.2 Households' optimization problem

Analogous to experts' optimization problem, households have objective to maximize expected discounted lifetime utility or value function, except that they are unconstrained in the choice

[20] The terms $e^{\mu^p+\sigma\sigma^p}, e^{\sigma}, e^{-\frac{1}{2}(\sigma+\sigma^p)^2}$ represent geometric mean, i.e. median(GM), geometric standard deviation(GSD) of market losses, and convexity correction term(CC) from Ito's lemma. Therefore, a risk measure can be rewritten as $1-\frac{GM}{(GSD)^{\Phi^{-1}(\alpha)}}CC$, and intuitively tells how "far" the distorted diffusion process is from a random walk, where randomness captures prices we cannot control.

[21] One of the most intuitive and rememberable interpretations of signal-to-noise ratio is Kahneman's "favorite equation": *success = talent + luck, great success = a little more talent + a lot of luck*

[22] As a reference, if the market price is Gaussian, 95 % of price changes are within two standard deviations. In comparison, the 2008 market downturn was about 10 standard deviations surprise.

[23] Markowitz (1952)'s a measure of the expected return is the mean and of risk is variance.

of capital. Therefore, they only face endogenous net worth evolution similar to experts'

$$d\underline{n}_t = (r_t\underline{n}_t + a\underline{k}_t)dt + p_t\underline{k}_t(\mu_t^p + \sigma\sigma_t^p - r_t)dt - \underline{c}_t dt + p_t\underline{k}_t(\sigma + \sigma_t^p)dW_t. \tag{13}$$

In particular, households choose consumption $\underline{c}_t$ and capital demand $\underline{k}_t$ in order to maximize value function subject to net worth evolution

$$\begin{aligned} V(n_0) = \max_{\underline{c}_t,\underline{k}_t} \quad & E\left[\int_0^\infty e^{-rt}\log \underline{c}_t dt\right] \\ s.t. \quad & d\underline{n}_t = (r_t\underline{n}_t + a\underline{k}_t)dt + p_t\underline{k}_t(\mu_t^p + \sigma\sigma_t^p - r_t)dt - \underline{c}_t dt + p_t\underline{k}_t(\sigma + \sigma_t^p)dW_t. \end{aligned} \tag{14}$$

Although households are unconstrained in capital demand, they experience market gains and losses if decided to hold some capital and produce. They insure against downside market risk by providing risk-free debt to experts. Alternatively, experts partially insure against market losses by holding sufficient net worth.

3.3 Equilibrium

Equilibrium *Given the initial endowment of capital* $(k_0, \underline{k}_0)$, *an equilibrium is a collection of allocations* $(k_t, \underline{k}_t, c_t, \underline{c}_t, n_t, \underline{n}_t,)$ *and a price process* p_t *such that*

(i) experts maximization problem is solved,

(ii) households maximization problem is solved,

(iii) markets for output and capital clear.

Let us first solve for the households' optimization problem, which we assume is stationary (i.e., tradeoffs only depend on the state variable n_t) and apply the dynamic programming approach. Households' *Hamilton-Jakobi-Bellman* equation is

$$rV(\underline{n}_t) = \max_{\underline{c}_t,\underline{k}_t} \quad \log \underline{c}_t + V'(\underline{n}_t)[a\underline{k}_t + r_t\underline{n}_t + p_t\underline{k}_t(\mu_t^p + \sigma\sigma_t^p - r_t) - \underline{c}_t] + \frac{1}{2}V''(n)(\sigma + \sigma_t^p)^2 p_t^2 \underline{k}_t^2 \tag{15}$$

where $V(\underline{n}_t)$ denotes households' value function. The mathematical derivation of the HJB equation is the consequence of Ito's lemma. The intuition comes from the fact that Brownian motion has a lot of volatility even in small intervals, which contributes to the drift whenever $V(\cdot)$ is convex or concave. In economic interpretation, the right-hand side terms denominate instantaneous utility, gains or losses from the drift and gains or losses from the volatility of net worth (expected and unexpected balance sheet gains and losses), while the left-hand side term represents an instantaneous value function.

The first order condition for consumption implies the Euler equation, optimal consumption is such that the marginal utility of consumption is exactly the same as the marginal utility of wealth

$$\frac{1}{\underline{c}_t} = V'(\underline{n}_t). \tag{16}$$

The first order condition for capital gives the asset pricing equation if households choose to hold capital

$$\frac{a}{p_t} + \mu_t^p + \sigma\sigma_t^p = r_t + \frac{-V''(\underline{n}_t)(\sigma + \sigma_t^p)^2 p_t \underline{k}_t}{V'(\underline{n}_t)}. \tag{17}$$

The expected return on capital is equal to the risk-free interest rate plus the risk premium, i.e. the equity premium equals the risk premium. Substituting first order conditions into HJB equation gives the second order linear differential equation whose solution is given by the following proposition.

Proposition 2. *Households' optimal consumption and capital rule are linear in wealth and the value function is given by*

$$c_t(n_t) = rn_t$$

$$\underline{k}_t(n_t) = \frac{\frac{a}{p_t} + \mu_t^p + \sigma\sigma_t^p - r_t}{p_t(\sigma + \sigma_t^p)^2}\underline{n}_t$$

$$V(\underline{n}_t) = \frac{1}{r}\log(r\underline{n}_t) + \frac{1}{r^2}\left(r_t - r + \frac{(\frac{a}{p_t} + \mu_t^p + \sigma\sigma_t^p - r_t)^2}{2(\sigma + \sigma_t^p)^2}\right).$$

PROOF. See the Appendix A. □

Let us now solve for experts' maximization problem. Experts' *Hamilton-Jakobi-Bellman* equation is

$$\rho V(n_t) = \max_{c_t, k_t} \quad \log c_t + V'(n_t)[ak_t + r_t n_t + p_t k_t(\mu_t^p + \sigma\sigma_t^p - r_t) - c_t] + \frac{1}{2}V''(n)(\sigma + \sigma_t^p)^2 p_t^2 k_t + \xi\left[n_t - p_t k_t M\right]. \tag{18}$$

The optimal policies for consumption and capital demand and a value function are characterized by two optimality conditions and the Lagrange multiplier on the capital requirement constraint

$$\frac{1}{c_t} = V'(n_t) \tag{19}$$

$$\frac{a}{p_t} + \mu_t^p + \sigma\sigma_t^p = r_t + \frac{-V''(n_t)(\sigma + \sigma_t^p)^2 p_t k_t + \xi p_t M}{V'(n_t)} \tag{20}$$

$$\xi(n_t - p_t k_t M) = 0. \tag{21}$$

As we will see below, a tightening of the borrowing constraint affects the external finance premium, as well as the equity premium that experts earn in equilibrium. A higher capital buffer is beneficial if market losses are realized albeit entails an opportunity cost if gains are realized since these funds could have been invested in the more productive capital.[24] Further, since households are unconstrained, the equity premium they earn on capital equals the risk pre-

[24]The Lagrange multiplier is the shadow cost of internal funding while $r_t - \xi_t$ is the measure of the external financing premium.

mium, i.e. the Sharpe ratio multiplied by the variance of expected consumption growth.[25] On the other hand, experts receive additional compensation because the borrowing constraint directly affects risk preferences in terms of the marginal utility of wealth. In effect, households and experts do not earn the same equity premium. We summarize the optimal consumption and capital choices of experts and value function in the preceding proposition.

Proposition 3. *Optimal consumption and capital rules and the value function of experts are given by*

$$c(n_t) = \rho n_t$$

$$k_t = \frac{n_t}{p_t M_t}$$

$$V(n_t) = \frac{1}{\rho}\log(\rho n_t) + \frac{1}{\rho^2}\left(r_t - \rho + \frac{1}{M_t}(\frac{a}{p_t} + \mu_t^p + \sigma\sigma_t^p - r_t) - \frac{(\sigma + \sigma_t^p)^2}{2M_t^2} \right)$$

PROOF. See the Appendix A. □

Let $\psi_t = \frac{k_t}{K_t}$ denote experts' share of capital, $\eta_t = \frac{N_t}{p_t K_t}$ denote experts' wealth share, and K_t denotes aggregate capital supply in the economy. We can summarize the Markov equilibrium in the state variable η_t, where all variables are functions of the current value of η_t. In the preceding proposition, we derive the law motion of the experts' wealth share, which we further use to obtain the wealth share probability distribution and systemic risk in terms of the experts' probability of being undercapitalized.

Proposition 4. *Experts' wealth share η_t evolves as*

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta dt + \sigma_t^\eta dW_t \tag{22}$$

with drift $\mu_t^\eta = \dfrac{1}{M_t}\dfrac{a}{p_t} - \rho + (\dfrac{1}{M_t} - 1)(\mu_t^p + \sigma\sigma_t^p - r_t - (\sigma + \sigma_t^p)^2)$ *and volatility* $\sigma_t^\eta = (\dfrac{1}{M_t} - 1)(\sigma + \sigma_t^p)$.

PROOF. See the Appendix A. □

We have three market clearing conditions as follows. First, aggregate capital demand in the economy is the sum of experts' or households' capital demand and equals the exogenous capital supply. Second, aggregate wealth is equal to the market value of the aggregate capital. Third, since there are no physical capital investments in the economy aggregate output equals aggregate consumption. Finally, because short selling of capital is not allowed, $\psi_t = \min(1, \dfrac{\eta_t}{M(\eta_t)})$. If short selling was an admissible strategy, ψ_t could be greater than one.

- *Market clearing for capital*

$$k_t + \underline{k}_t = K_t, \quad i.e. \quad \psi_t + (1 - \psi_t) = 1 \tag{23}$$

[25] Because of log utility, the variance of expected consumption growth is equal to the variance of net worth growth.

- *Aggregate wealth*

$$N_t + \underline{N}_t = p_t(k_t + \underline{k_t}) \tag{24}$$

$$\eta_t + (1 - \eta_t) = 1 \tag{25}$$

- *Market clearing condition for output*

$$\rho N_t + r\underline{N}_t = ak_t + \underline{ak}_t \quad i.e.$$

$$p_t(\rho\eta_t + r(1-\eta_t)) = a\psi_t + \underline{a}(1-\psi_t)$$

In summary, we obtain the system of ordinary differential and algebraic equations with the endogenous state variable $\eta_t \in [0,1]$ and boundary conditions

1. Risk-based capital requirement (macroprudential regulation)

$$M_t = 1 - \mathrm{e}^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau + \Phi^{-1}(\alpha)(\sigma+\sigma_t^p)\sqrt{\tau}} \tag{26}$$

2. Marked to market balance sheet

$$\sigma_t^\eta = (\frac{1}{M_t} - 1)(\sigma + \sigma_t^p) \tag{27}$$

3. Asset market feedback

$$\sigma_t^p = \sigma_t^\eta \eta_t \frac{p'(\eta_t)}{p(\eta_t)} \tag{28}$$

4. Marked to market balance sheet

$$\mu_t^\eta = \frac{1}{M_t}\frac{a}{p(\eta_t)} - \rho + (\frac{1}{M_t} - 1)(\mu_t^p + \sigma\sigma_t^p - r_t) + (1 - \frac{1}{M_t})(\sigma + \sigma_t^p)^2 \tag{29}$$

5. Asset market feedback

$$\mu_t^p = \frac{p'(\eta_t)}{p(\eta_t)}\mu_t^\eta \eta_t + \frac{1}{2}\frac{p''(\eta_t)}{p(\eta_t)}(\sigma_t^\eta \eta_t)^2 \tag{30}$$

6. Fire-sale discount(market clearing for output)

$$p(\eta_t)\,(\rho\eta_t + r(1-\eta_t)) = a\psi(\eta_t) + \underline{a}(1 - \psi(\eta_t)) \tag{31}$$

7. Fire sales / households' risk premium

$$\frac{\underline{a}}{p(\eta_t)} + \mu_t^p + \sigma\sigma_t^p - r_t = \frac{1-\psi(\eta_t)}{1-\eta_t}(\sigma + \sigma_t^p)^2 \tag{32}$$

8. Margin-CAPM /experts' equity premium

$$\frac{a}{p(\eta_t)} + \mu_t^p + \sigma\sigma_t^p - r_t = \frac{1}{M}(\sigma + \sigma_t^p)^2 + \xi_t p(\eta_t) M_t \tag{33}$$

9. Wealth share probability distribution / systemic risk (Kolmogorov forward equation)

$$0 = -\mu_t^\eta(\eta_t)\eta_t f(\eta_t) + \frac{1}{2}\frac{\partial}{\partial\eta}((\sigma_t^\eta(\eta_t)\eta_t)^2 f(\eta_t)) \tag{34}$$

with the boundary condition at $\eta = 0$

$$p(0) = \frac{a}{r} \tag{35}$$

In Appendix B, we provide insights on prominent transmission channels of exogenous capital quality shock captured by the above equations. In particular, we consider balance sheet channel, fire sales, and asset price feedback, margin-based asset pricing channel and how regulation alters these channels captured by the above equations. Before we proceed to the main implications of macroprudential regulation, we emphasize several features and limitations of equilibrium and summarize key ideas that drive dynamics in equilibrium. First, it is crucial to assert that we solve the model in the steady state, which can be seen from the absence of time derivative in the Kolmogorov equation. Doing so we abstract from transition dynamics and we can only say how wealth distribution, prices, and systemic risk behave in the long run. Even if macroprudential regulation may be important in the long run, the impact might be more pronounced in the short run. Transition dynamics are important in their own right since they answer the relevant question: if a financial sector undertakes a macroprudential regulatory reform such as the change from Basel II to Basel III to reduce systemic risk, how long would it take until favorable results become evident and which factors accelerate or delay the transition. Another simplifying assumption is that capital supply follows a Brownian motion as a parsimonious way to capture transitory productivity shocks. The effect of the persistence of productivity shock on capital requirements and systemic risk is presently beyond the scope of the paper. Third, a strong solution of the model involves solving stochastic differential equations instead of ordinary differential equations. This desirable feature matters for the quantitative analysis as it would be possible to use parameter estimation methods for stochastic differential equations instead of calibrating parameters.[26]

Proposition 5. *Stationary wealth share distribution is given by*

$$f(\eta) = C\frac{e^{2\int_0^\eta \frac{\mu^\eta(\eta')}{\sigma^\eta(\eta')^2\eta'}d\eta'}}{\sigma^\eta(\eta)^2\eta^2}, \tag{36}$$

[26]The general form of the stochastic differential equation is $dX_t = \mu(X_t)dt + \sigma(X_t)dW_t$, where $\{W_t\}_{t\geq 0}$ is a standard Brownian motion (in our model $X_t = \eta_t$ and we solve for $\mu^\eta(\eta_t), \mu^p(\eta_t), \sigma^\eta(\eta_t)$ and $\sigma^p(\eta_t)$). A strong solution of this stochastic differential equation with initial condition $x \in \mathbb{R}$ is an adapted process $X_t = X_t^x$ with continuous paths such that for all $t \geq 0$, $X_t = x + \int_0^t \mu(X_s)ds + \int_0^t \sigma(X_t)dW_t$, which involves solving the stochastic integral. This could be done if our system of ordinary differential equations had an explicit solution, i.e. functional form for $\mu^\eta(\eta_t)$ and $\sigma^\eta(\eta_t)$) and not needed to be solved numerically(implicitly) as in the following section.

where C is the normalizing constant.

PROOF. See the Appendix A. □

We are interested in localizing the peaks of the stationary distribution which occurs when the exponent reaches maximum, i.e. when the cumulative ratio of expected and unexpected balance sheet gains and losses reaches the maximum value [27]. Maximum value can be attained at the interior point or the two boundary points. In Brunnermeier and Sannikov (2014), the distinction between the crisis and normal times is captured by two-peak stationary distribution reaching peaks at two boundaries: one around zero and the second at the state where experts expected wealth share equals zero. Moreover, in their model, the systemic risk in terms of the probability that experts are undercapitalized endogenously emerges because experts do not insure against aggregate risk. In contrast, in financial accelerator Bernanke et al. (1999) type of models, predicts a stable stationary normal distribution around the steady state and systemic risk is negligible($f(0) \approx 0$). As we will see in what follows, equilibrium here features stationary distribution which is "in between" two models depending on the choice of prudential instrument. In technical terms, macroprudential regulation channel (salience of market losses) affects the salience of balance sheet gains and losses, and thereby wealth share probability distribution.

In a nutshell, two main ideas underpin results. The first idea is that experts' risk attitudes in choice and risk attitudes in pricing are independent due to the binding borrowing constraint. Investor's choices are described as risk-averse if he chooses a sure payoff (gain or loss) over uncertain payoff. In this respect, risk attitude in choice reveals investor's willingness to invest in the risky asset. Risk attitude in pricing instead conveys the investor's willingness to pay or sell for the risky asset. A risk-averse agent is willing to accept a certainty equivalent that is lower than the expected value of uncertain outcome to lock in sure payoff, in effect paying the positive premium to avoid uncertainty. As agents can invest in the risky capital and risk-free deposits and markets are incomplete, risk attitude in choice determines capital allocation. Instead, risk attitude in pricing governs the equity premium. For instance, with log utility and unconstrained choice of capital, households' risk attitudes in choice and pricing coincide with the equity premium. In contrast, the binding borrowing constraint separates risk attitudes in the pricing of aggregate risk and market tail risk for experts. The equity premium can be decomposed into the risk premium and salience loss premium. Measured market losses affect the equity premium, which is not the case in pure consumption-based models. Moreover, when the financial sector is regulated, experts' capital allocation depends on regulatory future market losses, and therefore risk-taking deviates from the equity premium.[28]

The second idea is that regulators depart from the expected utility theory, the standard theory of choice under risk. When determining the capital buffer to be held against risky assets and accordingly the risk-taking of financial institutions, regulators attach values to gains and

[27]The cumulative ratio of expected and unexpected balance sheet gains and losses is equal to $\int_{\eta}^{\eta^*} \frac{\mu^{\eta}(\eta')}{\sigma^{\eta}(\eta')^2\eta'} d\eta'$.

[28] The idea that regulatory risk measure may be related to risk-taking of the financial sector has been recognized previously in Adrian and Shin (2010).

losses rather than wealth. Second, a small probability bad outcome of the risky asset is salient for regulators. In particular, they overweight VaR_α loss and assign zero weight to gains and residual losses, regardless of whether these are likely or unlikely, to begin with. Therefore, regulators' risk perception distorts decision weights attached to uncertain outcomes. Regulators assess the probability of unlikely VaR_α event correctly, however they overweight this unlikely event in their decisions.[29] In effect, they perceive the risky asset to be worse than it actually is, as VaR_α is de facto certainty equivalent for tail market losses. As a result, the salience of unexpected market losses determines the financial sector risky asset allocation, regardless of the risk-free asset payoff.

4 Equilibrium dynamics

In the benchmark model we solve the system for the following set of parameters

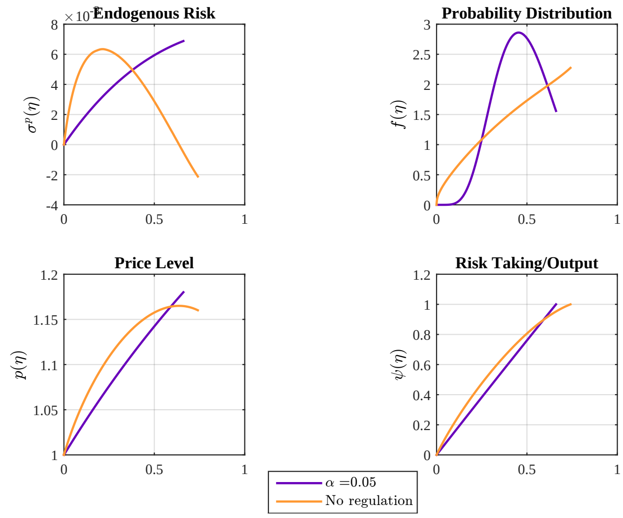
$$a = 0.055, \quad \underline{a} = 0.04, \quad r = 0.04, \quad \rho = 0.05, \quad \sigma = 0.1, \quad \tau = 30, \quad \alpha = 0.05.$$

Figure 3a, 3b 4a depict the optimal values of output, asset price, systemic and endogenous risk, as well as various risk attitudes and compensations as a function of intermediaries' liquid wealth. We can interpret the decrease in experts wealth share as an adverse balance sheet shock such as shocks to the market valuation of risky assets, comparable to a subprime shock. We consider two regulatory regimes. In the first case, prudential authority is non-existent and the financial sector does not insure against market losses. In the second regime, market losses are measured by value at risk and capital requirement constraint is binding.

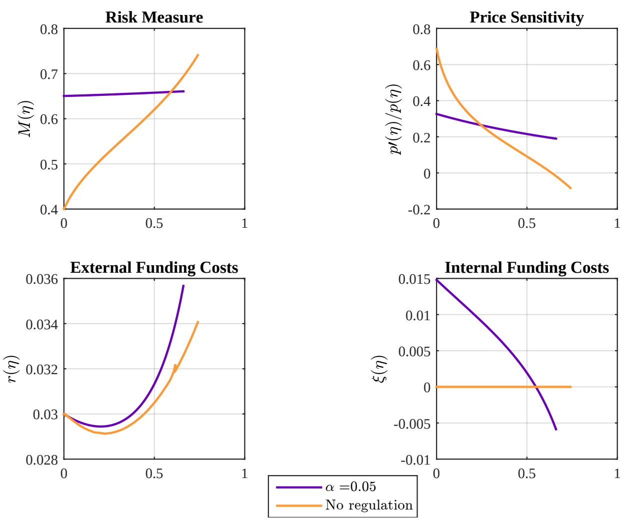
Different responses at different levels of experts liquid wealth share are apparent. Under VaR regulation, the financial sector's the tightness of the capital requirement constraint endogenously determines a twofold preference pattern: risk aversion and risk seeking in market losses. Two thresholds such that $\xi(\eta) = 0$ and $\psi(\eta) = 1$ indicate levels of liquid funds at which experts stop being constrained by the capital requirements and stop investing in capital. They then determine frequencies of a financial and real boom and bust episodes assigned by the endogenous probability distribution, while the levels of price and output indicate the magnitude of booms and busts.

In the first region $\eta \in [0, \eta^\xi]$, experts are risk-averse in expected market losses. The econ-

[29] In expected utility, utilities of gains and losses are allowed to differ only in sign but not in magnitude, and the decision weights equal probability of the outcome. Two regulators' assumptions are similar to the main two assumptions in the prospect theory, namely the focus on gains and losses and distortion of decision weights. However, regulators define losses as the difference between expected future wealth and current wealth, while in Tversky and Kahneman (1992) losses are defined as current wealth minus past period wealth. Second, in their model decision weights are distorted in a way that events with small probabilities are overweighted(*possibility effect*) while events with high probabilities are underweighted(*certainty effect*) and decision weights are smooth. Improbable outcomes are given higher weight than their probability justifies while almost certain outcomes are given less weight then justified by their probability. With VaR_α as a risk measure, weights are discontinuous. Losses with probability smaller than α(losses above VaR_α threshold) are given zero weight, whereas in Tversky and Kahneman (1992) these are overweighted.



(a) systemic risk measures

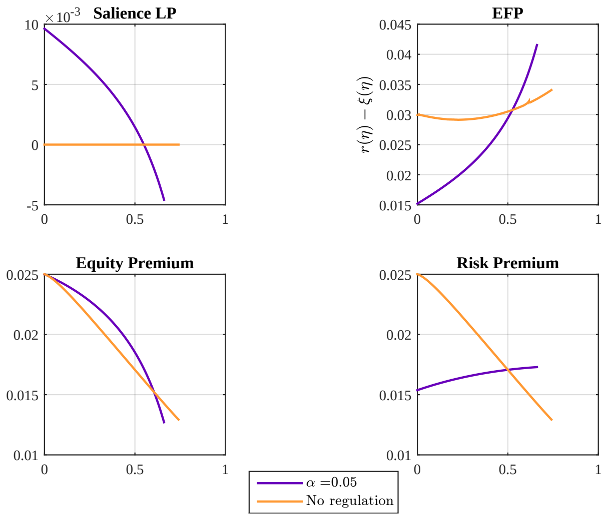


(b) risk attitudes and measures

Figure 3: Equilibrium with macroprudential VaR

omy is characterized with lower economic growth where most of the capital is allocated to households, and endogenous risk and capital requirements are lower. At the first threshold η^{ξ}, experts are indifferent between taking the gamble or a sure VaR_{α} market loss. We may interpret this as a certainty equivalent in market tail losses: as far as risk measurement decisions are concerned, experts behave as if future tail losses were certain and equal to VaR_{α}. Below this threshold, experts are risk-averse in the pricing of market losses and demand positive salience loss premium to avoid the uncertainty in the realization of market losses.

In the second region $\eta \in [\eta^{\xi}, \eta^{\psi}]$, experts are risk-seeking in future expected losses and salience loss premium is negative. During this period of higher asset price growth, more capital is allocated to experts so the output growth rate is higher than in the first region. At the second threshold η^{ψ}, the short sale constraint which prevents experts from shorting capital from households puts the bound on the level of liquid funds experts can attain by truncating



(a) risk compensations

Figure 4: Equilibrium with macroprudential VaR

the probability distribution from the right and therefore restricting the risk-seeking in losses region.[30]

By taking a narrow perspective, salience loss premium reveals differences between regulators' and banks' estimates of market risk and required capital to absorb losses. In a more broader view, it reflects the alignment of regulatory and financial sector objectives. While regulators aim to prevent insolvency, banks aim to maximize shareholders value. The main question is why there are variations in this premium? One explanation of why experts demand positive salience loss premium in busts is because the downside potential of the risky asset is salient. This reasoning is in line with salience theory of Bordalo et al. (2013b), where the downside is likely to be salient if market losses are higher than market gains, or less likely to occur.

By the same token, the upside is likely to be salient if gains are higher or infrequent. In this case, the financial sector requires a negative premium. The salience of small probability market losses elicits risk aversion or risk-seeking attitudes. These variations in banks' perception of the size of market losses and capital buffer to absorb losses become apparent when we recall that VaR_α acts as certainty equivalent for tail losses. In our model the intuition for a twofold pattern is straightforward: experts think about losses measured by regulators, focus on the upside where market losses are lower than VaR and downside where market losses are higher than VaR, and overweight losses which drive their attention.

When choosing a risk measure, regulators need to evaluate the bigger picture at a level beyond the financial sector because it is the externalities of endogenous risk-taking that matter. Regulators need to control the risk-taking, but they also need to consider the systemic risk that arises when choices and attitudes of both agents interact in equilibrium. Figure 3a, top right panel helps us understand the effect of macroprudential VaR_α policy on systemic risk. We find

[30]If the short sale of capital was allowed, the financial sector could switch from external debt financing to internal financing to purchase capital with a positive probability.($\eta^\psi = 1$ and $\psi > 1$).

that macroprudential VaR can mitigate systemic risk since endogenous probability of experts being undercapitalized is close to zero.[31] Under VaR, the probability distribution peaks in the risk-averse region. Without regulation, the financial sector is more likely to be undercapitalized or overcapitalized in comparison to VaR.

While systemic risk is generally considered to be the primary reason for the supervision and regulation of the financial sector, the systemic risk might have its roots in experts' risk-seeking attitudes. When experts focus on the upside potential of the risky asset and overestimate their ability to avoid market losses, they underestimate exposure to financial(asset price) cycle. This hope to avoid large losses can lead to bailout expectations and moral hazard. The tradeoff between bailout expectations formed in booms and actual bailouts in downturn lies the core of an adequate choice of prudential instrument.

An alternative measure of systemic risk which we suggest is the probability that the salience loss premium is positive. The reason for such proposition is because when the financial sector reports insufficient equity to absorb market losses, this can lead to costly government interventions such as capital injection and recapitalization as evidenced by the aftermath of the 2008 market downturn. Because risk attitudes arise endogenously, experts demand positive salience loss premium when they perceive market losses to be higher than assessed and measured by regulators.

We now look at how prudential VaR affects risk-taking.[32] Banks' willingness to take on risk operates through two main channels Freixas et al. (2015). Preference channel refers to risk attitudes of financial institutions as the primary cause of investing in the risky asset. These can be rational such as habit-formation, or behavioral such as over-optimism or neglecting tail risk. According to agency channel, explicit or implicit government guarantees such as deposit insurance and bailout expectations can influence banks' investment decisions. Mainly they can create moral hazard since implicit guarantees mean that losses are shared between the nonfinancial and financial sector while gains are internalized by the latter.

Figure 3a in right bottom panel summarizes optimal risk-taking behavior in the presence and absence of bank regulation. We observe that banks engage in more risk when the financial sector is unregulated. One possible reason for this result is that the financial sector does not internalize the systemic risk costs of its risky loans. This view is consistent with moral hazard literature and agency channel as part of insolvency costs is borne by lenders rather than banks themselves. However, the reason why they do so lies in their risk attitudes. Intuitively, banks build up more risk when market gains and losses are proportionally valuated. This is the case with expected utility decision-makers who focus equally on the upside and downside potential of the risky asset. Attaching same decision weights to favorable and unfavorable outcomes makes banks risk-neutral in the pricing of market losses. Banks do not overweight or underweight market losses. Consequently, *undistorted* perception of the risky asset leads banks to be insufficiently insured: they do not internalize the systemic externalities of default. In other words, preference channel elicits agency channel and is responsible for a higher willingness to

[31] $f(0) \approx 0$.

[32] Financial sector risk-taking is equal to $\psi(\eta) = \frac{\eta}{M(\eta)}$ when experts are regulated.

invest in the risky asset in the absence of prudential regulation.

Another important finding is that risk-taking remains procyclical when banks are required to maintain capital ratios. To a certain extent, this prediction can be reconciled with the preference channel. Although macroprudential regulation can counteract banks' risk-taking incentives that exacerbate systemic risk, it is risk attitudes of regulators that determine banks' capacity to take risks. As mentioned earlier, in their risk assessments regulators disproportionally pay attention to small probability loss. Relying on *VaR* to absorb losses leads to countercyclical capital requirements. As capital ratios rise faster when banks' equity is low, risk-taking is less procyclical in busts, and more procyclical in booms.

Our model sheds light on the external finance premium introduced in financial accelerator channel by Bernanke and Gertler (1990). The external finance premium is the key variable providing evidence for the existence of financial frictions and credit market imperfections, which in turn amplify shocks arising from the real sector. Measured by the difference between the costs of external financing and costs of internal financing such as retained earnings, it is in general positive, reflecting the lender's cost of monitoring and evaluating borrowers' risky prospects. The financial accelerator also predicts that the external financing premium rises in recessions, as borrowers' are more likely to default. We find support for positive external finance premium, so two sources of financing are imperfect substitutes. Importantly, our result suggests that credit market imperfections are less severe when the financial sector is regulated.[33] In particular, higher systemic risk in unregulated regime is associated with higher external financing premium. This result does not follow from asymmetric information between a borrower (experts) and a lender (households), which is the most prevalent explanation for the existence of the external finance premium. The main empirically testable hypothesis we derive is that the external finance premium reveals the *divergence* in lenders' and borrowers' attitudes towards downside market risk, rather than expected loss premium per se. This divergence in attitudes itself may arise due to heterogeneity in information processing of the financial and nonfinancial sector, or design of prudential policy instrument.[34]

To elaborate more on this point, from a broader perspective market tail risk which regulators aspire to control and systemic risk are downside tail risks. VaR_α risk measure quantifies downside *price* risk potential, while systemic risk measure reflects the downside *balance sheet* potential of the financial sector. Since explicit government guarantees for demand deposits are nonexistent in this economy, households may demand deposit insurance premium which incorporates compensation for systemic risk or expected market losses of the financial sector.[35] If households require only systemic risk premium, we would expect external financing to be higher when no regulation is imposed. The opposite is the case, and external financing is more

[33] See Figure 4a, top right graph.

[34] One notable measure of investors' risk attitudes and the external finance premium is the excess bond premium introduced by Gilchrist and Zakrajšek (2012). Authors demonstrate that the EBP has higher predictive power for recession risk compared to expected default risk.

[35] Acharya et al. (2010) suggest that the systemic risk in the financial sector is a key determinant of efficient deposit insurance premiums.

expensive with regulation.[36] On the other hand, if households require compensation only for expected losses, we would expect two interest rate paths to cross at some level of intermediaries wealth, which also does not hold. To the extent that capital requirement constraint always binds, the interest rate possibly reflects the lenders' trust in the macroprudential risk measure to absorb market losses. In this way, the compensation that lender requires beyond expected losses reveals the willingness of lenders to bear default risk. Similarly, without bailout expectations, experts may also demand compensation for expected losses. In the regulatory regime, expert exhibit either risk-seeking or risk-averse behavior and the binding constraint introduces the cost of holding internal funds. By contrast, when experts are unregulated, they are risk-neutral in market losses and salience loss premium is zero.

The financial accelerator also predicts that borrowers with the weak balance sheet in terms of current cash flows pay a higher premium for external finance. If borrower's liquidity is impaired, this likely increases the risk to the lender and creates incentives for a lender to demand a higher risk adjustment. We find that external financing premium actually increases with η, which may be an immediate implication of the divergence of risk attitudes. Intangible characteristics of a lender and a borrower such as risk attitudes might be the prevailing determinants of the external finance premium instead of tangible cash flows. The intuition is straightforward and plausible: when thinking about investing in deposits, households evaluate borrower's preferences for risk and his ability to repay debt. As expert's liquidity improves and η increases, they shift from risk-aversion to risk-seeking and hope to avoid losses. With higher expected losses and borrower's misbehavior, creditors' trust deteriorates. In summary, risk attitudes in the pricing of market losses seem to be responsible for the properties of the external finance premium, while risk attitudes in choice govern risk-taking.

One can look at two regimes to shed light on possible mechanism underlying endogenous risk and fire sales. For example, is fire-sale discount larger or smaller when banks are subject to prudential regulation? Does market risk management amplify endogenous risk? If both market participants are unconstrained, endogenous risk reflects changes in asset prices due to portfolio adjustments in response to precautionary motives. With risk management policies, this risk arises in response to the news about regulatory market losses. *VaR* has the potential to destabilize price dynamics when the upside potential of the risky asset is salient. In a period of banks' distress and impaired liquidity, however, *VaR* can dampen endogenous risk. Perhaps more important, when both agents are unconstrained, the endogenous risk is significantly larger when banks are distressed (Figure 3a top left panel).

Nonetheless, it is interesting to see volatility dynamics through the lens of fire sales. Fire sales arise when borrowers are forced to liquidate assets after adverse balance sheet shock. At the same time, potential buyers are marginal investors (unproductive households) who value capital less than natural investors (productive experts), so capital is sold at a fire-sale discount. Consistent with Brunnermeier and Sannikov (2014), in the absence of a prudential instrument, fire sales produce a negative volatility spiral in which selling capital makes prices more volatile.

[36]See Figure 3b, left bottom graph.

This happens because recovery from the undercapitalized region is slower. By contrast, with prudential regulation volatility declines because insolvency periods are less frequent. Regulators can, therefore, stop adverse spiral between lower asset prices and higher endogenous risk.

However, with risk management "flight to quality" is intensified, as fire-sale discount tends to increase relative to the baseline case of no regulation.[37] Fire sale discount is a proxy for market illiquidity, the difference between the first-best price of capital and its liquidation value. When marginal investors hold all capital in the economy, they value capital solely by fundamentals, by their preferences and productivity.[38] For this reason, the fire-sale discount depends on how agents react to news about measured regulatory losses in booms.

The mechanism underlying endogenous risk and fire sales may be related to the way marginal and natural investors update their beliefs in light of incoming news about market losses. Marginal investors may overreact and assign an excessive weight to future outcomes that have become more likely with recent risk assessments. For example, after a period of tighter regulation, households may become complacent about systemic risk. In such an environment, the arrival of unfavorable news about future market losses may lead marginal investors to revise disproportionally their assessment of systemic risk and overweight the probability of a crisis. On the other hand, experts underweight crisis probability in booms because they underestimate their exposure to market losses.

Similar behavior has been observed during the recent crisis. In the run-up to 2008 market downturn, financial institutions struggled to determine the degree of their exposure to potential losses on securitized assets (MBS), which further destabilized financial markets and generated fire sales of MBS (Mizen (2008)). This reasoning implies that investors' information processing mechanism can amplify or mitigate volatility in market prices, even in the absence of significant changes in economic fundamentals.

Finally, we examine precautionary saving behavior of two sectors. The key idea of precautionary saving is that agents consume less and save more today when facing greater future income uncertainty or when the borrowing constraint might bind in the future. In general, prudence or future borrowing constraints are two motives for additional savings in order to take precaution against possible adverse realizations of the future income or cash flows.[39] If the agent is prudent the pain(i.e increase in marginal utility) from bad consumption state being realized is larger than the gain(i.e decrease in marginal utility) from the good state being realized. Hence, a rise in future income uncertainty leads to a decline in current consumption and an increase in current savings. Similarly, when agents face borrowing constraints, they fear to get several successive adverse income realizations which make the constraint binding and force them to consume current income without the ability to smooth consumption. To

[37] See Figure 3a, bottom left panel.

[38] The boundary condition at $\eta = 0$ is $p(0) = \frac{a}{r}$.

[39] The agent is prudent if the marginal utility is convex, while risk aversion refers to decreasing marginal utility, i.e. value function is concave. Prudence measures the agent's propensity to prepare and forearm oneself in the face of uncertainty, while risk aversion measures how much agent dislikes and wants to avoid the uncertainty (Kimball (1990)).

prevent this disliked situation, they self-insure by increasing savings in order to smooth consumption through income shocks. While a priori prudence and borrowing constraints may be relevant motives for households and experts to increase saving, regulators fear future large market losses being realized which push experts to the undercapitalized region without the ability of monetary policy to re-capitalize balance sheet of experts. For this reason, regulators take precaution by increasing capital requirements. We, therefore, compare consumption smoothing incentives across agents and in different regulatory regimes, when the financial sector is unconstrained or constrained in the choice of capital in Figure 4a.

Our first finding is that households' behavior is close to precautionary savings models in Carroll and Kimball (1996) when experts are undercapitalized and to the permanent income hypothesis when experts are sufficiently capitalized. Two saving options are available to smooth consumption, investing in risky capital or risk-free deposits. In busts when households are forced to hold most of the capital, their income volatility is high. In this context, precautionary savings implies higher wealth accumulation by increasing equity premium and deposit demand. Conversely, in booms when experts hold most of the capital, households' precautionary motive for income uncertainty subsides as their future income is less volatile. In essence, households are prudent and sufficiently patient so higher uncertainty is associated with more savings in a risk-free asset. The second finding is that salience of small probability market losses leads to *higher* prudence and more saving of the nonfinancial sector. As a result, households demand higher compensation as they hold larger share of capital when the financial sector is regulated.

In regulatory regime, the key insight is that saliency of large market losses causes a divergence in precautionary saving motives for future income uncertainty of the financial and nonfinancial sector. This finding is reflected in the procyclical risk premium and the countercyclical equity premium. With macroprudential regulation, two risks can generate precaution and self-insurance, market downside risk and future income uncertainty. While future tail risk is partially insured with VaR_α, the aggregate risk is uninsured. Since borrowing constraint is always binding, experts do not fear it may bind in the future and do not self-insure against that possibility. Instead, the financial sector partially internalizes the regulator's fear of large market losses and exhibits fear of future income volatility. This divergence occurs because risk assessments of *future* tail losses and *current* borrowing constraint generate a buffer-stock "saving" behavior of experts. As in Carroll (1997) buffer-savers target cash at hand or level of liquid funds so there is an upper limit for precautionary saving.[40] Prudence dominates if wealth is below the target while impatience dominates if wealth is above the target. If current cash is below the target experts will accumulate liquid funds, and conversely, deplete liquid funds if cash level is above the target. Intermediaries insure more against aggregate risk than future market losses as risk premium is higher than salience loss premium. In summary, the financial

[40] The target level of liquid wealth is defined as the level which remains constant in expectation, where liquid funds stop growing and cash flow process becomes a martingale, i.e. previously defined level η such that $\mu^\eta(\eta) = 0$. This is the point where the marginal value of saving and the marginal value of consumption of an extra unit of net worth are equal.

sector is prudent over future income risk, prudent over future market losses when the downside is salient, and imprudent when the upside is salient.
In the following sections, we aim to answer two questions. How does an economy with macroprudential VaR_α policy instrument respond to adverse shocks such as an increase in borrowing costs, a credit crunch or an increase in uncertainty? What are the implications of different macroprudential policies on financial frictions and systemic risk?

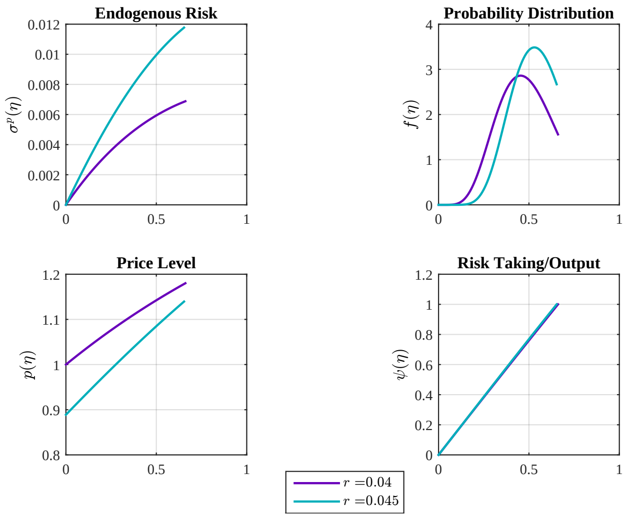
4.1 Sudden increase in borrowing costs

One of the great challenges regulators are facing is how to prevent variations in financing conditions to affect financial stability. Suppose the economy is in a steady-state with a stable distribution of borrowing and lending positions and financing costs. In this section, a sudden shock hits the financial system and borrowing gets harder in terms of tighter borrowing costs, such that external financing becomes more expensive. The opposite is a decrease in lending standards, which lowers the borrowing costs. We model the interest rate jump as an increase in households impatience rate r. Productive capacities in the economy stay the same.

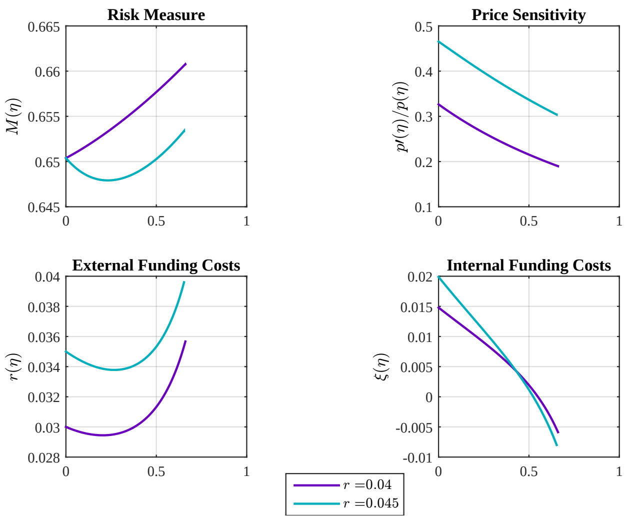
The prudential policy is unable to protect banks against forced liquidation of the risky asset. Regulators respond to impaired external funding by decreasing capital requirements and preventing risk-taking. In turn, this leads to "flight to quality" and selling capital to households. Figure 3a shows that difference in the valuation of capital between marginal and natural investors expands. Fire sale discount rises because the elevated impatience leads to the lower liquidation value of capital.

Consistent with Brunnermeier and Sannikov (2014) where experts are unregulated, borrowing frictions depress prices and investment. At the same time, absent regulation in Brunnermeier and Sannikov (2014), a rise in borrowing costs tend to stabilize the equilibrium. It leads to lower endogenous risk and lower crisis probability. Under prudential VaR, an increase in borrowing costs destabilizes price but alleviates systemic risk. Therefore, this instrument can manage crisis anticipations. It is, however, inefficient policy instrument to combat endogenous risk as prices become more sensitive and volatile.

Heightened impatience of lenders also give rise to financial market imperfections. Two effects contribute to an increase in the external finance premium. First, there is a mechanical effect, as borrowing costs increase since households have a higher rate of time preference. Second, tightening of external financing costs leads to tightening of internal financing costs in busts, which decreases the premium. Experts become more risk-averse in the pricing of market losses when the downside is salient and more risk-seeking when the upside potential of the capital is salient. Tightening of the external financing conditions makes it more difficult for experts to obtain outside liquidity, which forces them to demand higher risk and the equity premium. Internal funds become more valuable despite becoming more costly. Heightened impatience of lenders triggers a precautionary motive for income uncertainty, and the financial sector accumulates more liquid wealth to mitigate the increase in borrowing costs.



(a) measures of systemic risk



(b) risk attitudes

Figure 5: Increase in borrowing costs with macroprudential VaR

4.2 Credit crunch

How does an economy adjust to the liquidity shortage in the financial system? In this section, we examine the response of our economy to a credit crunch. A term credit crunch has been used to describe two things: a decline in credit availability in a financial system due to a shortage of liquidity, and decline in the credit availability for consumers and entrepreneurs (Mizen (2008)). Our analysis focuses on the former, the curtailment of the credit supply in response to a decline in the value of bank capital. We do so by increasing banks' impatience rate of ρ such that they consume a larger part of their liquid funds. In this respect, tightening of borrowing costs and credit crunch are adverse shocks to the outside and inside liquidity of the financial sector.

Figure 8a and 8b summarize the impact of internal liquidity shock. The immediate effect is that experts value capital less in states where they manage all capital in the economy, which

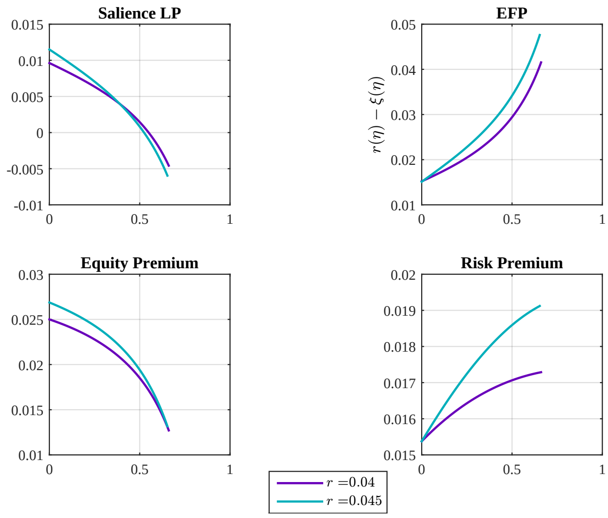


Figure 6: risk compensations

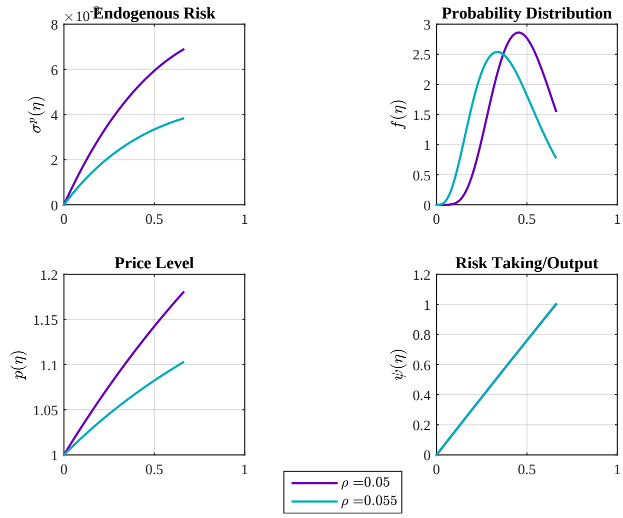
Figure 7: Increase in borrowing costs with macroprudential VaR

decreases fire-sale discount.

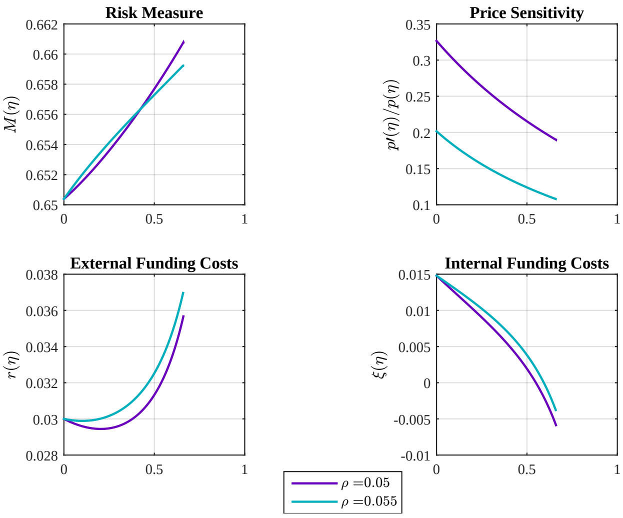
In response to bank capital shock, three effects combined generate the incentives to deplete the buffer-stock level of liquid wealth. First, the marginal rate of substitution between current and future consumption increases, so the opportunity cost of holding liquid assets rises. The first indirect effect is that the marginal value of saving for future income uncertainty declines as future income is less uncertain. Two effects together generate incentives to deplete liquid assets. The second indirect effect is that the marginal value of saving for market losses increases, which encourages intermediaries to save more by accumulating equity. Taken all together, an adverse inside liquidity shortage leads to the anticipation of the systemic crises.

Prudential policy, however, does not produce negative spillovers to the real economy. The investment rate slightly increases and more capital is allocated to experts. Consistent with Drehmann et al. (2010), countercyclical capital requirements can alleviate a credit crunch. Regulators respond to an environment with liquidity shortage by raising capital ratios. In so doing, households overweight measured losses and demand higher deposit insurance premium, which raises the interest rate. A similar response is observed of experts as they become more risk-averse in the pricing of market losses. Two agents overweight measured losses to a comparable degree with experts becoming more risk-averse, which leads to a small decrease in the external financing premium. And, crucially, the economy as a whole underestimates intermediaries' exposure to the financial cycle, which decrease endogenous risk.

Comparing shock propagation under two liquidity shortage scenarios, macroprudential VaR can either reduce the ex-post crisis anticipation or amplification, but it cannot control both. As emphasized by Freixas et al. (2015), higher ex-ante vulnerabilities such as price volatility lead to higher ex-post systemic risk. However, a buildup of imbalances by itself is not a sufficient condition to generate crisis anticipations. Ex-ante financial vulnerabilities coupled with a



(a) systemic risk measures



(b) risk attitudes

Figure 8: Credit crunch with macroprudential VaR

sudden balance sheet shock give rise to systemic risk.

4.3 Uncertainty shock

Uncertainty has received substantial attention as an important factor in shaping the severity and the longevity of the great recession. For instance, Stock and Watson (2012) suggest that the financial and uncertainty shocks are principal contributors to output decline during the great recession. Similarly, Bloom et al. (2018) propose that uncertainty shocks are new shocks which drive business cycles. That said, we aim to investigate does regulation amplify or dampen fundamental cash flow risk?

Suppose the exogenous risk rises, which we model as an increase in σ. In response to elevated uncertainty, regulators instantaneously boost capital buffers to reflect an environment

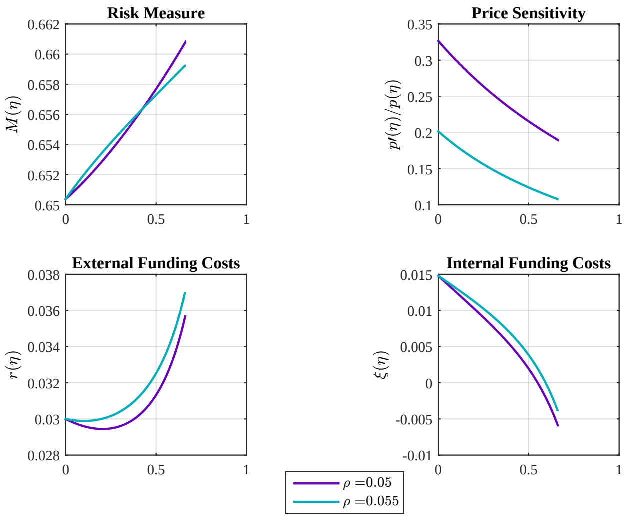


Figure 9: risk compensations

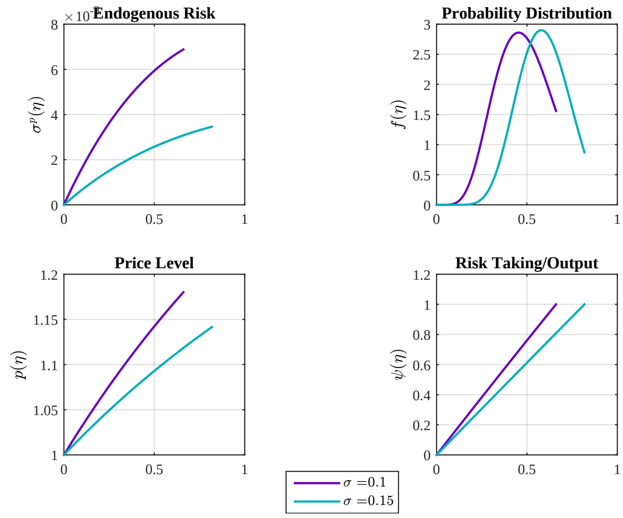
Figure 10: Credit crunch with macroprudential *VaR*

prone to higher risk. In turn, higher capital ratios force experts to hold more liquid internal funds on their balance sheet. Also, higher uncertainty makes experts more prudent, which increases the risk premium on productive assets and depresses the price. This response is in line with Carroll (1997), where higher future income uncertainty is associated with higher prudence and buffer stock saving. Overall, the precautionary motives both for higher future income uncertainty and market tail losses help explain why banks accumulate more equity. Similarly, the interest rate drops sharply, reflecting the households' strong precautionary incentives to save after the uncertainty has been realized.

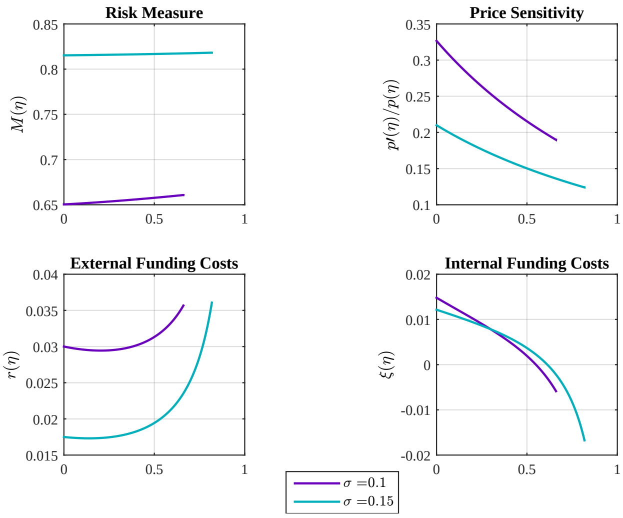
The fundamental question is whether risk management matters when it comes to stabilizing the economy. The impact of increased uncertainty on macroprudential policy effectiveness in combating risks is notable. Regarding system stability, endogenous and systemic risk are dampened in the face of higher uncertainty. The key to this aspect of policy effectiveness is that a rise in uncertainty makes experts more cautious in their perceptions of measured future market losses in booms, while the economy as a whole is less cautious about systemic risk. In the first case, the financial sector becomes less risk-seeking. In the second case, three agents underestimate the probability of a systemic crisis, which decreases endogenous risk.

However, a vigorous policy response attests regulators' inability to prevent capital reallocation in the economy. By reducing risk-taking of the financial sector, higher buffers impede growth prospects. In this way, the cost of focusing on worst-case market outcomes with *VaR* include the forgone output in times of economic expansions rather than in recessions.

The fact that the pricing and choices of three agents arise endogenously allow us to understand when financial vulnerabilities emerge in our economy. At the heart of the results lies the response of macroprudential *VaR* policy and the perceptions of the financial sector to potentially adverse shocks. Systemic risk manifests when the inside liquidity of the financial sector is



(a) systemic risk measures



(b) risk attitudes

Figure 11: Uncertainty shock with macroprudential VaR

impaired, and the financial sector becomes more risk-averse and less risk-seeking. Although in this case, intermediaries overweight their exposure to market losses, the economy as a whole underweights its exposure to systemic events. The necessary condition for the endogenous risk to manifest seems to have its roots in economic agents jointly overweighting its exposure to systemic events. Finally, fire sales do not emerge when the adverse shock arises from the banks' balance sheet. This may shed lights on whether severe recessions and fire sales result from widespread uncertainty shocks or funding conditions of the financial system. As consistent with Bloom et al. (2018), uncertainty shocks may be the key factor in shaping economic activity.

From a broader perspective, both the likelihood of a systemic crisis and fire sales are different measures of systemic risk. Overall, the main conclusion we derive is that the effectiveness of macroprudential VaR in combating adverse shocks is dependent on the type of shock and

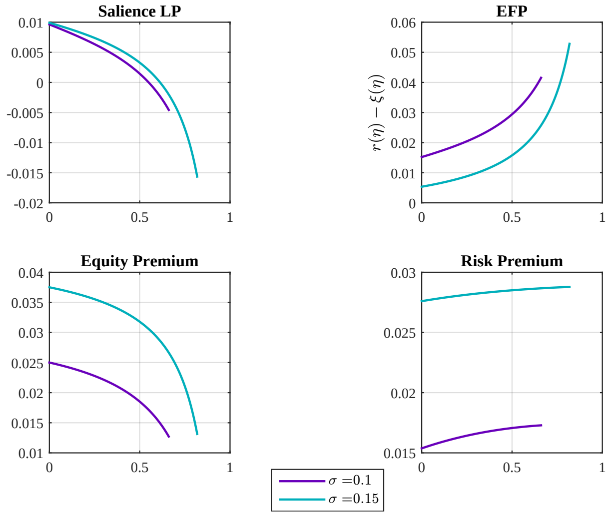


Figure 12: risk compensations

Figure 13: Uncertainty shock with macroprudential VaR

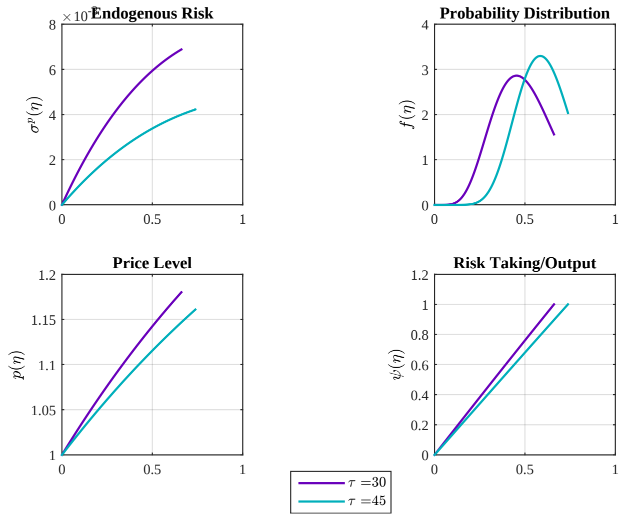
the measure of systemic risk. Qualitatively at least, we suggest that macroprudential VaR is more effective in controlling systemic and endogenous risk when banks are exposed to market risk rather than liquidity risk.

4.4 Financial regulation: longer-term and longer-shot macroprudential VaR

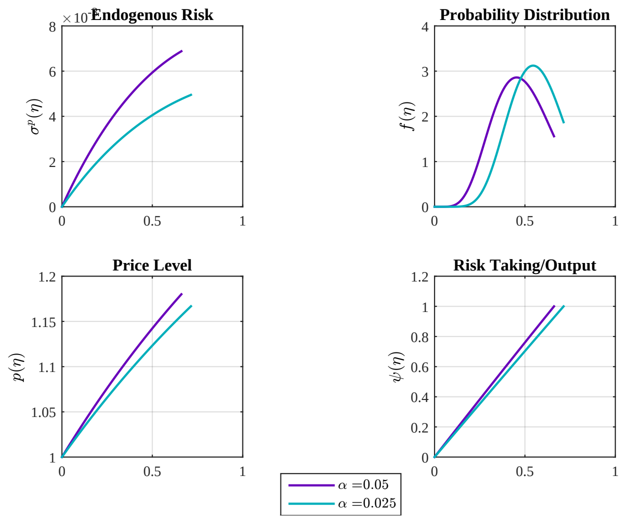
Notable attention has been given to the role of prudential tools to limit the probability and severity of the financial crises.[41] However, there has not yet been consensus on how to compare the effects of different regulatory frameworks such as macroprudential VaR_α in Basel II and III. Aforementioned issue is not present here, as we can directly compare different regulatory frameworks by specifying a different market risk measure. The ultimate objective of this comparison is evident: to understand the implications of macroprudential regulation on systemic risk. In the following, we aim to answer the question: what are the costs, benefits, and trade-offs which arise after adopting a tighter or a more accommodative macroprudential policy instrument?

First, let us suppose that macroprudential policy adopts a more forward-looking perspective and aims to quantify longer-term market tail losses to prevent systemic risk. In this macroprudential policy setting, regulators increase the time horizon τ for which market losses are quantified (Figure 14a and 15a). At first glance, the benefits of long-term risk management include alleviating all financial imbalances and frictions. In particular, endogenous and systemic risk decline, as well as the fire-sale discount and the external financing premium. The implications initially seem surprising, but these result from several conjunctions. First, capital requirements are higher since the goal of regulators is market loss absorbency over a longer time horizon. Higher capital requirements mean that the financial sector is less leveraged

[41]See for instance Acharya (2009) for a survey.



(a) systemic risk measures

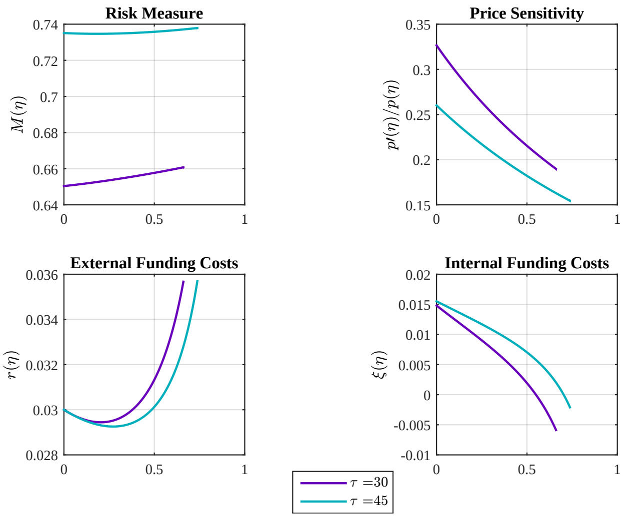


(b) systemic risk measures

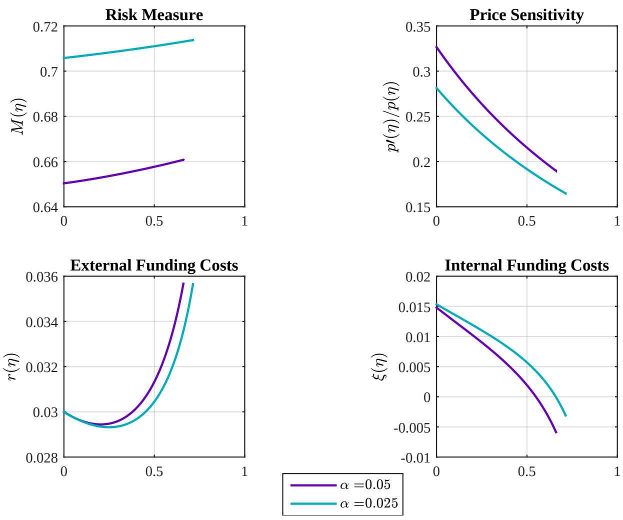
Figure 14: Longer-term and longer-shot macroprudential *VaR*

in the new equilibrium, which deflates crisis anticipation. Second, the economy as a whole underweights downside balance sheet risk, which in turn decreases endogenous risk. Third, households become less risk-averse in pricing measured market losses, leading to a reduction in the external financing premium. As regulators are primarily concerned with risk management, the crucial disadvantage of longer-term policy is its spillover to the real sector. With more forward-looking policy, risk-taking of the financial sector decreases,resulting in subdued growth prospects.Capital reallocation to the unproductive sector after new regulation suggests a causal relationship from the prudential regulation to the real economic activity.

Suppose now tighter regulation of the financial sector is implemented by allowing for a lower probability of market tail risk(Figure 14b and 15b). In principle, we can interpret *VaR* risk measure as a long-shot bet(a lottery with a small probability of large losses). Beyond detailed mechanism, a closer look at both longer-term policy and longer-shot policy reveals an intu-



(a) risk attitudes



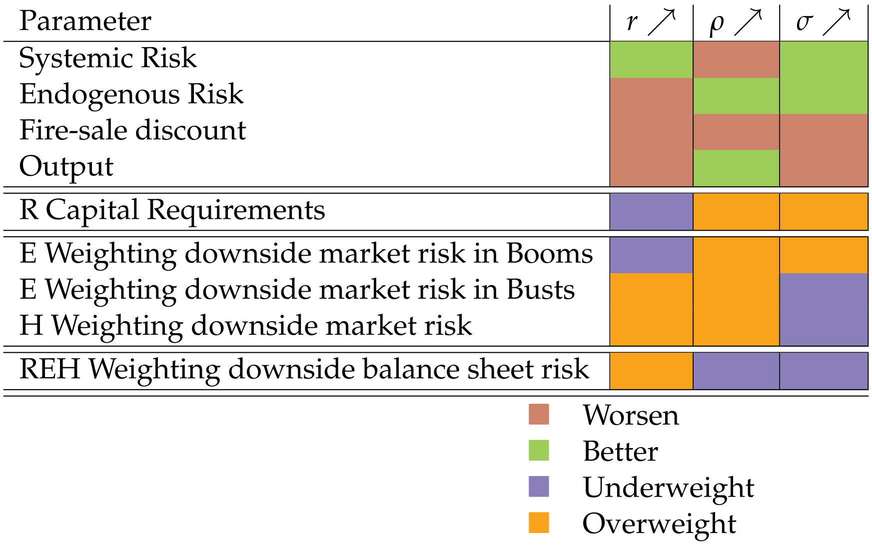
(b) risk attitudes

Figure 15: Longer-term and longer-shot macroprudential VaR

itive result: a decrease in portfolio adjustments is essentially the same as demanding lower loss probability. Conversely, macroprudential VaR with higher loss probability entails same costs and benefits as if regulators focus on shorter-term losses. To put this logic in context, if regulators take for granted an increasingly larger use of derivatives and securitization in day-to-day financial trading which allows for shorter-term portfolio adjustments without adjusting the regulation, regulators' inaction entails the same costs*as if* they allowed for higher loss probability. De facto, there are hidden costs of regulatory inaction.

In summary, we present costs and benefits of macroprudential VaR capital requirements in Table 1. In the following section, we investigate the effectiveness of the alternative prudential regulation, where market losses are quantified by spectral risk measures instead of VaR.

Table 1: Cost and Benefit Analysis of Macroprudential VaR Policy Instrument



5 Spectral risk measures

As we have seen in the previous section, with a tighter macroprudential VaR the main tradeoff which arises is between lower output and lower systemic risk. Moreover, VaR seem not the be the adequate choice of a risk measure when an economy is facing uncertainty shock as growth prospects become subdued. In this section, we aim to answer the question: can regulatory risk measure be designed to account for financial imbalances and severity of recessions and magnitude of forgone output? The theory of spectral risk measures may offer a possible solution. Knowing that VaR and Expected Shortfall cannot capture upside risks and the continuity of the risk attitudes of regulators, we focus on spectral risk measures instead (Acerbi (2002)). Therefore, the goal of this section is to construct a general framework for risk measures that allow analyzing simultaneously upside and downside risks. As before, we asses the efficiency of the proposed regulation.

Ultimately, the main challenge is how to represent risk attitudes of regulators. The key idea is to devise a framework consistent with salience and psychology of attention in Bordalo et al. (2013b) and Tversky and Kahneman (1992), where the decision-maker evaluates lotteries by overweighting the most salient states. The probability weighting function constitutes "the local thinking" and captures the strength of the decision maker's focus on salient states. In this way, it proxies for the agent's ability to pay attention to multiple events or outcomes. For instance, with VaR regulators' behavior is consistent with extreme local thinking as they focus on a single rare event and banks' vulnerability to unlikely market losses.

In terms of risk attitudes, we specify the probability weighting function in three ways. The important implication is that changing which market outcomes drive regulators' attention lead to quantitatively different assessments of market risk. First, we consider risk management policy where regulators are concerned with banks' exposure to tail events and insure against it. To do so, we use Wang (2000)'s distortion function which transforms the probability distribution by overweighting the risk in the tails.[42] Second, we construct decision weights in the spirit of Tversky and Kahneman (1992) where prudential authority overweights small probabilities and underweight almost certain market outcomes. In this regime, regulators are mainly concerned about how banks are exposed to losses in stress scenarios but also hopeful about losses that arise in favorable periods. Third, we consider the opposite case where regulators underweight small probabilities and overweight high probability outcomes instead. Regulators neither evaluate much favorable or unfavorable scenarios, but are concerned about average losses that arise in normal times. In this respect, what constitutes the most salient state for regulators changes across different regimes.

In technical terms, the probability weighting function for Wang's distortion measure is equal to

$$g_W(p) = \mathrm{e}^{-\frac{b^2}{2} + b\Phi^{-1}(p)} \tag{37}$$

and the risk spectrum is such that $g(p) = G'(p)$ is equal to[43]

$$G_W(p) = \Phi(\Phi^{-1}(p) - b) \tag{38}$$

where b is a constant to be chosen. To obtain plausible values for b, we estimate the coefficient b from the asset pricing equation from the main text by the general method of moments presented in Appendix C. This estimation procedure leads to an approximate value of b about -1.3, which we further use in comparison of prudential policies. Figure 16 implies that for negative values of b banks overweight their exposure to market losses, while for positive they underestimate potential losses. On the other hand, the risk spectrum and the probability weighting function in Tversky and Kahneman (1992) are equal to[44]

$$g_{KT}(p) = ap^2 + bp^2 + c \tag{39}$$

and

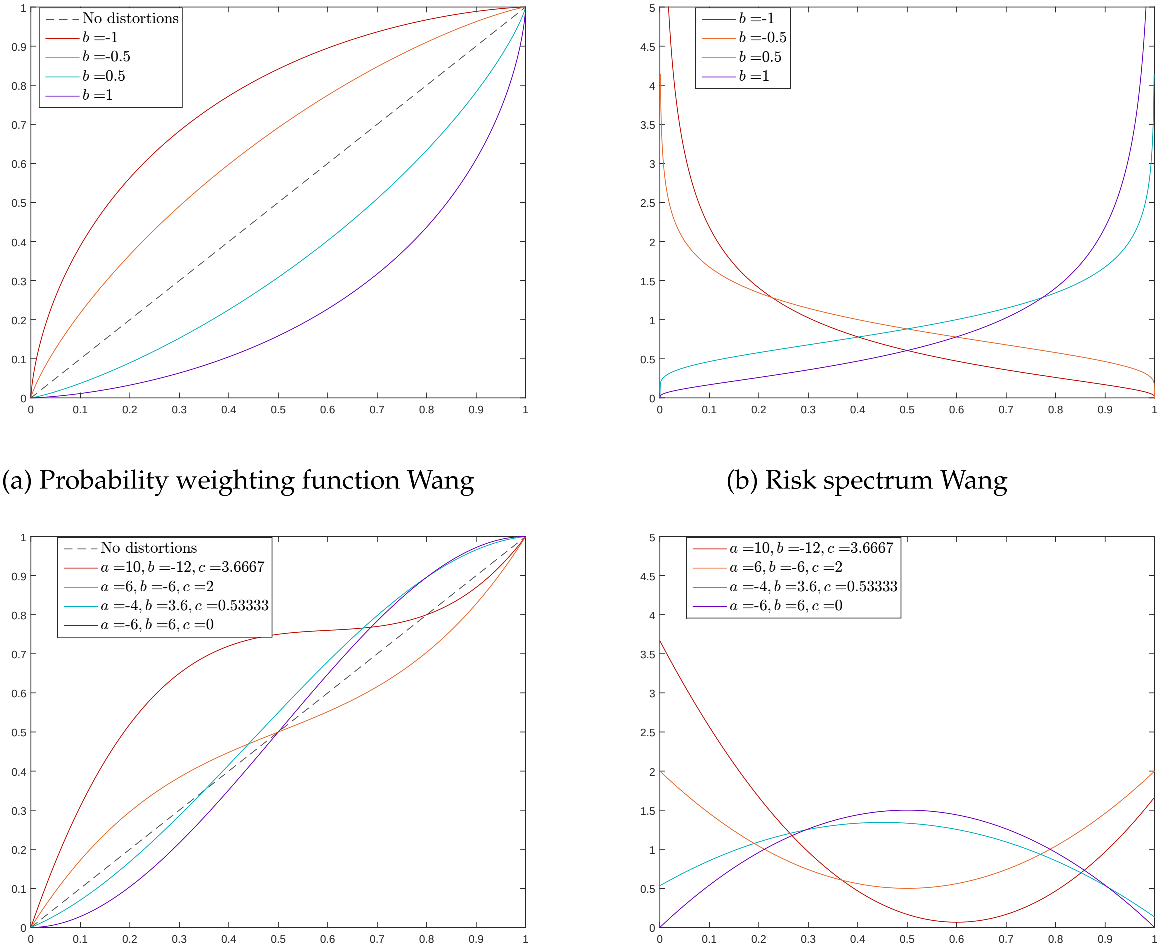
$$G_{KT}(p) = \frac{a}{3}p^3 + \frac{b}{2}p^2 + cp \tag{40}$$

for positive a. The key characteristics of the prospect theory of Tversky and Kahneman (1992), namely the possibility effect (overweighting of small probabilities) and certainty effect (underweighting of large probabilities) are captured by an inverse S-shaped probability weighting function. It is concave for low probabilities and convex for high probabilities. This shape

[42] A distortion function transforms the probability distribution of the losses to another probability distribution by re-weighting the original distribution and is just an alternative notation for decision weights or probability weighting function in Tversky and Kahneman (1992)

[43] It is straightforward to prove this, where as before $\Phi(\cdot)$ is the cdf and $\phi(\cdot)$ the pdf of the standard normal distribution. $G'(p) = \frac{\partial\Phi(\Phi^{-1}(p)-b)}{\partial p} = \phi(\Phi^{-1}(p) - b)\frac{\partial(\Phi^{-1}(p)-b)}{\partial p} = \frac{\phi(\Phi^{-1}(p)-b)}{\phi(\Phi^{-1}(p))} = \mathrm{e}^{-\frac{b^2}{2}+b\Phi^{-1}(p)}$. The second equality follows from the chain rule of derivatives, the third from the derivative of inverse function and the fourth from the definition of $\phi(\cdot)$ and canceling common terms.

[44] In Tversky and Kahneman (1992), decision weights are equal to $G(p) = \frac{p^\gamma}{(p^\gamma+(1-p)^\gamma)^{\frac{1}{\gamma}}}$. The literature on spectral risk measures has predominantly used the decision maker's utility function as a guidance to construct the probability weighting function. Some choices of a weighting function can be found in Guegan and Hassani (2015).



(a) Probability weighting function Wang

(b) Risk spectrum Wang

(c) Probability weighting function KT

(d) Risk spectrum KT

Figure 16: Probability weighting functions and risk spectrum

conveys a psychological mechanism underlying probability distortions in the prospect theory, which they call *diminishing sensitivity*: the decision-maker is less sensitive to changes in probability as they move away from two reference points : 0 and 1. The first reference point indicates outcomes which will certainly *not* happen, while the second portrays events that will certainly happen. The risk spectrum when a is positive conveys this intuition clearly with two peaks at the ends of the interval; distortions are more pronounced at ends than in the middle of the distribution.

Perhaps not overly highlighted by prospect theory, diminishing sensitivity can also occur with a single reference point such as status quo. This is the case when a is negative, and the risk spectrum peaks in the middle in Figure 16. For simplicity and intuitive appeal, we call this anti-KT risk measure since the probability weighting function is S-shaped. In contrast to previous risk measure, the decision-makers' attention is drawn to the intermediate reference point instead of extreme ones. In the case of Wang's distortion with $b = -1.3$, the attention of the regulators is drawn to the left extreme reference point.

Spectral risk measures for Wang's transformation and KT are

$$
\begin{aligned}
M_W &= \int_0^1 g_W(p) F^{-1}(p) dp \\
&= \int_0^1 \mathrm{e}^{-b^2 + b\Phi^{-1}(p)} (1 - \mathrm{e}^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau + \Phi^{-1}(p)(\sigma+\sigma_t^p)\sqrt{\tau}}) dp \\
&= 1 - \mathrm{e}^{(\mu^p + \sigma\sigma^p)\tau + b(\sigma+\sigma^p)\sqrt{\tau}}
\end{aligned}
$$

and

$$
\begin{aligned}
M_{KT} &= \int_0^1 g_K T(p) F^{-1}(p) dp \\
&= \int_0^1 (ap^2 + bp^2 + c)(1 - \mathrm{e}^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau + \Phi^{-1}(p)(\sigma+\sigma_t^p)\sqrt{\tau}}) dp \\
&= 1 - \mathrm{e}^{(\mu^p + \sigma\sigma^p)\tau} \left(c + (b+a)\Phi(\frac{(\sigma+\sigma^p)\sqrt{\tau}}{\sqrt{2}}) - 2aT(\frac{(\sigma+\sigma^p)\sqrt{\tau}}{\sqrt{2}}, \frac{1}{\sqrt{3}}) \right)
\end{aligned}
$$

, where $T(\cdot,\cdot)$ is Owen's T function.[45] The proof for M_{KT} is presented in Appendix A.

In the following, we solve the steady equilibrium with three types of spectral risk measures. As previously mentioned, we set $b = -1.3$ for Wang's risk measure while for KT and anti-KT we choose parameters of the probability weighting function represented by the red and blue line in Figure 16d. The important constraint when choosing parameters for KT is that risk spectrum should have the minimum value at an interior point, while for anti-KT maximum value is attained at an interior point. We also asses the effectiveness of alternative macroprudential instruments in mitigating time and cross-sectional dimensions of systemic risk after an increase in borrowing costs, a credit crunch and an increase in uncertainty. Such a framework allows us to compare different prudential tools and compare their effects.

Table 2 summarizes how our model implies the qualitative choice of a risk measure suitable for different adverse shocks. From a systemic risk perspective, macroprudential instruments aim to address the time dimension and cross-sectional dimension of systemic risk (Borio (2014); Freixas et al. (2015)). The former reflects intermediaries' procyclical risk attitudes in pricing. As mentioned in the previous section, procyclicality manifests in a buildup of endogenous risk and may arise because of explicit and implicit incentives from prudential policy (agency channel) or due to risk attitudes of financial intermediaries (preference channel). The overreaching goal of regulators is to reduce procyclicality to limit the likelihood of systemic crises. The cross-sectional dimension reflects the distribution of the risk in the economy and can cause fire sales and spillovers to the real sector. Therefore, prudential policies aim to address these externalities and provide stabilization once the crises occur.

Several implications for regulators and risk-managers arise from the preceding findings. Figure 17b implies that focusing on banks' exposure to tail risk brings a lower endogenous

[45] It is easy to notice that both Wang's and KT risk measure are an expectation of log-normal variable multiplied by the distortion term ($\mathrm{e}^{(\mu_t^p + \sigma\sigma_t^p)\tau}$), which is exactly what a spectral risk measure is, an expectation of a random variable with respect to the distorted probability distribution.

Scenario	endogenous risk	systemic risk	fire-sale discount	output
Borrowing costs	KT, anti-KT	VaR, Wang	VaR, Wang	KT, anti-KT
Credit crunch	VaR, Wang	anti-KT	anti-KT	VaR, Wang
Uncertainty	VaR, Wang	VaR, Wang	KT	KT
Equilibrium	Wang	VaR	VaR	anti-KT

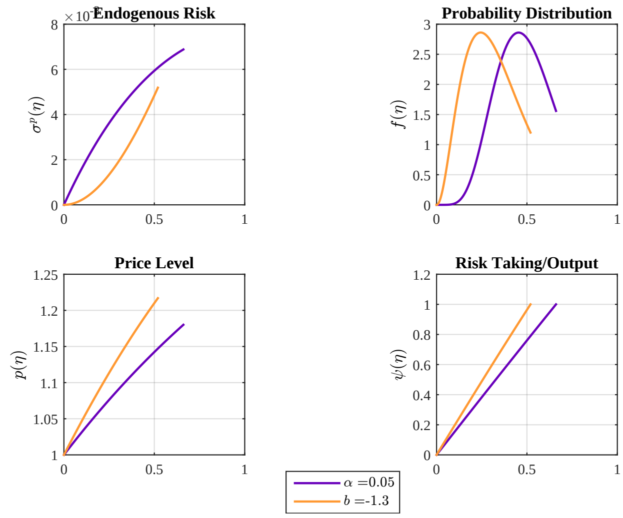
Table 2: Recommended Choice of a Macroprudential Risk Measure

risk, but a higher systemic risk and fire-sale discount in comparison to VaR. This finding may be a consequence of several transmission channels. First, lower capital requirements imply lower loss absorbency. Countercyclical capital requirements that rise in booms and fall in busts endogenously emerge. Interestingly, the Basel III regulatory framework implements countercyclical capital requirements, that also arise when assessing risk with VaR. Second, the financial sector remains risk-averse in busts and risk-seeking in booms. In quantitative terms, intermediaries become less risk-averse. Third, banks engage in more risk-taking for the same level of equity. These observations suggest that both the agency channel and preference channel are more active when market risk is quantified with Wang's risk measure. Given more accommodative regulatory measurement of market risk, we interpret the strengthening of two transmission channels as consistent with our prediction of mirroring of risk attitudes in pricing. As regulators become less risk-averse in the pricing of market risk with Wang's measure, financial intermediaries to a certain degree mirror regulators' perception of market risk. In turn, this may amplify systemic risk.

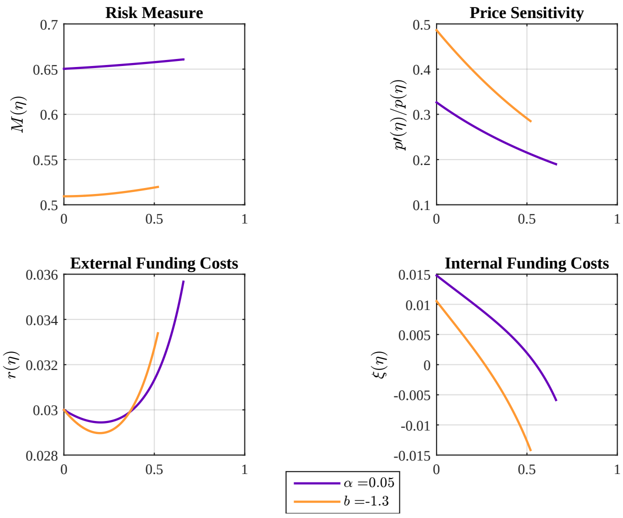
If regulators are fearful about tail losses but hopeful about small losses or divert attention to intermediate losses instead, these risk attitudes result in an even larger risk buildup. As Figure 18a and 18a illustrate, the major weakness of KT and anti-KT is that both measures exacerbate the time and cross-sectional dimension of systemic risk. In these two regimes, leverage acceleration is associated with a higher fire-sale discount and the likelihood of financial crises. Indeed, capital requirements are lower and procyclical in comparison to VaR and Wang. This means that higher loss absorbency(lower leverage) is available in downturns and lower in booms. The rapid and leveraged asset price boom has been the reliable precursor of financial crises and turned out to have more severe consequences throughout the history of these crises.[46] At the same time, financial intermediaries fail to internalize potential social costs of market losses as they become risk-seeking. Consequently, if the likelihood of a systemic crisis is the primary reason for the regulation of the banking industry, ex-ante systemic risk is best addressed when regulators only focus on tail losses.

If however, the main mission of regulators is to build a resilient financial system where institutions can absorb adverse shocks with minimum negative spillovers to economic and financial spheres, an adequate choice of a prudential tool is less than a straightforward answer. Even though alternative regulation leads to higher accumulation of systemic risk, ex-post crisis management is a crucial component when analyzing the strengths and weaknesses of the

[46]For instance, the housing and credit bubbles that preceded the great recession in 2008 are associated with a significantly larger contraction in economic activity than equity funded 2000 dot-com bubble.



(a) systemic risk measures

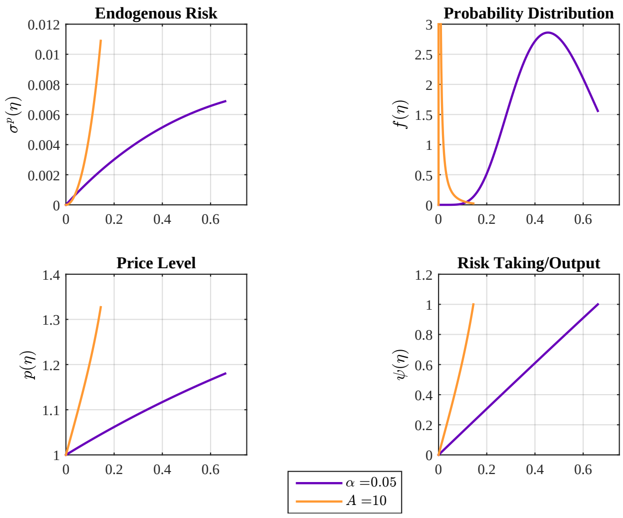


(b) risk attitudes

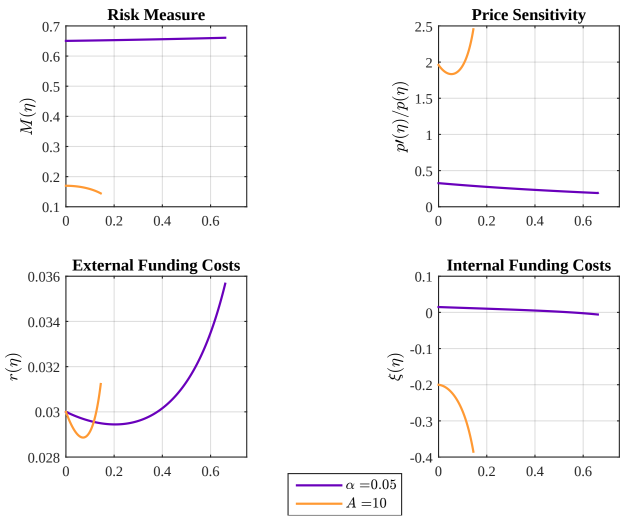
Figure 17: Equilibrium dynamics with macroprudential Wang and VaR

prudential framework. In this way, stress tests are a way to asses impact of capital requirements in various scenarios. Scenario analysis usually draws from historical stressful events such as 2000 dot.com bubble and 2008 market downturn, and estimates portfolio loss if similar circumstances are to be repeated. This type of sensitivity analysis quantifies impact of large movements in financial markets such as uncertainty or liquidity shocks so that vulnerabilities of prudential framework can be identified.

First, our results suggest there is very little efficiency gain if regulators solely pay attention to tail market risk. The same costs and benefits summarized in Table 1 for VaR also apply to Wang's risk measure (Table 5). In the 2008 crisis aftermath, the Basel III regulatory framework has adopted Expected Shortfall that measures average tail market losses as a new market risk measure in response to this particular deficiency of VaR. Relatedly, Gennaioli et al. (2012) argue that neglecting tail risk has important implications for systemic risk, and is crucial for un-



(a) systemic risk measures



(b) risk attitudes

Figure 18: Equilibrium dynamics with macroprudential KT and *VaR*

derstanding aspects of the 2008 great recession. Specifically, they suggest that when investors and financial intermediaries neglect downside tail risk because they cannot imagine worst-case outcomes during quiet times, systemic risk sharply increases. Exactly the opposite holds here, and the systemic risk rises when tail market losses are included. At first sight, we might expect that correcting the incentives of prudential policy by including tail losses is sufficient to correct weaknesses of *VaR* because intermediaries operate under binding regulation. Opposite to our priors, we find that even if regulators consider the likelihood of a severe decline in market prices, this risk attitude of regulators may not be sufficient to remove weaknesses of ex-post crisis management with *VaR*. Instead, we suggest that non-neglecting tail downside risk has limited benefits, and a way forward in the prudential framework may be the inclusion of upside risks.

Table 3: Cost and Benefit Analysis of Macroprudential *KT* Policy Instrument

Parameter	$r\nearrow$	$\rho\nearrow$	$\sigma\nearrow$
Systemic Risk			
Endogenous Risk			
Fire-sale discount			
Output			
R Capital Requirements			
E Weighting downside market risk in Booms and Busts			
H Weighting downside market risk			
REH Weighting downside balance sheet risk			

Table 4: Cost and Benefit Analysis of Macroprudential $anti-KT$ Policy Instrument

Parameter	$r\nearrow$	$\rho\nearrow$	$\sigma\nearrow$
Systemic Risk			
Endogenous Risk			
Fire-sale discount			
Output			
R Capital Requirements			
E Weighting downside market risk in Booms and Busts			
H Weighting downside market risk			
REH Weighting downside balance sheet risk			

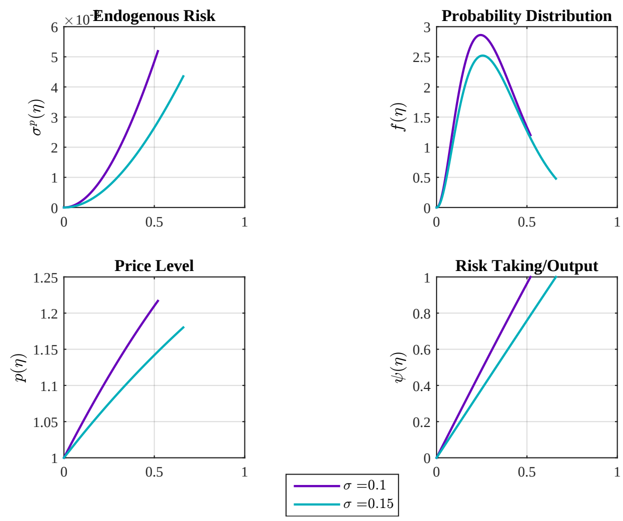
Table 5: Cost and Benefit Analysis of Macroprudential *Wang* Policy Instrument

Parameter	$r\nearrow$	$\rho\nearrow$	$\sigma\nearrow$
Systemic Risk			
Endogenous Risk			
Fire-sale discount			
Output			
R Capital Requirements			
E Weighting downside market risk in Booms and Busts			
H Weighting downside market risk			
REH Weighting downside balance sheet risk			

For example, when uncertainty shock arises, it proxies for correlated risk that makes cash flows of both sectors more volatile. An important question we want to answer is: which risk measure can account for the magnitude of forgone output after an adverse uncertainty shock?

In the previous section, we have seen that the main transmission channel of a substantial uncertainty shock with VaR is a sharp reduction in credit causing a decline in aggregate output and fire sales. Similarly, focusing on the worst possible outcome with Wang generates fire sales (Figure **??**). By contrast, KT measure not only prevents fire sales, but the uncertainty shock stipulates fire *buys* and economic expansion as banks are willing to buy capital at a higher price (Figure **??**).

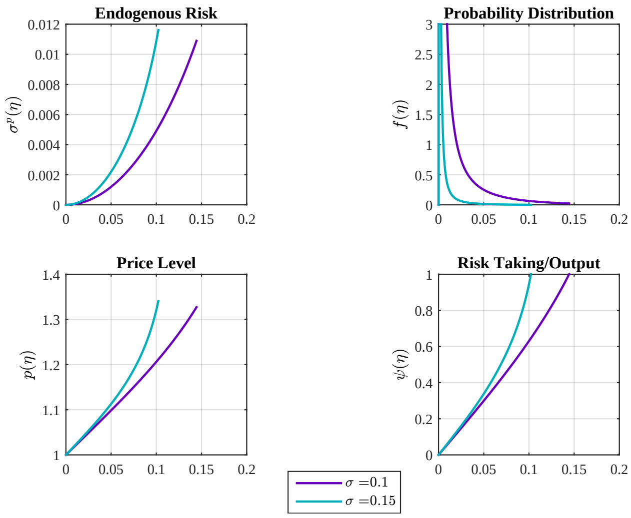
In a run-up to the 2008 financial crisis, selling mortgage-backed securities severely depressed their price, as both regulators and buyers and sellers were uncertain how to evaluate risks associated with these assets. Perhaps debatable, our result suggests that downward-adjusting all three market participants' perception of what will most likely happen when quantifying market risk, can boost both output and prices. By focusing solely on extreme market losses or by speculating what might happen in the face of higher uncertainty, the other three risk measures can neither prevent output drop nor fire sales in asset prices. Under KT regulation, however, fire buys lead to amplification and slower recovery from the undercapitalized region. This result occurs because the prudential policy responds by reducing capital requirements but in aggregate three agents overweight the exposure of intermediaries balance sheet to market losses.



(a) systemic risk measures

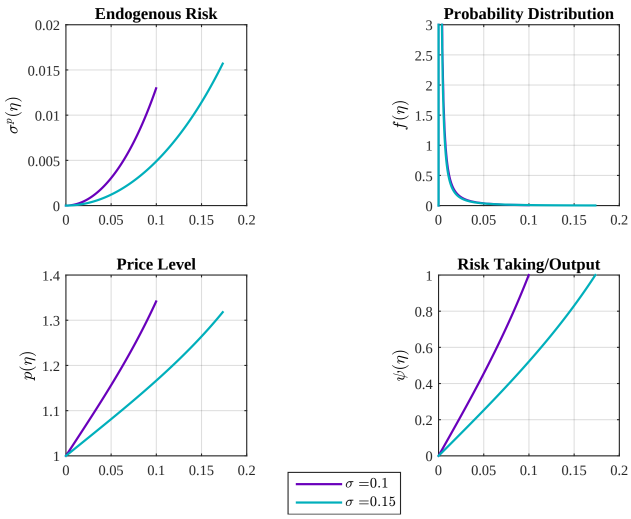
Figure 19: Uncertainty shock with macroprudential Wang

In managing the liquidity risk of financial institutions, regulators should quantify market risk by aligning with risk attitudes of the sector from which the shock originates to stabilize systemic risk and reduce the price discount. For example, when lenders become impatient and external financing tightens, regulators should focus exclusively on tail risk. Similarly, if banks become impatient and bank equity is impaired, admitting upside state realizations of the risky asset is beneficial when the shock arises from the financial sector. Conversely, reverse mirroring risk attitudes reduces endogenous risk and increases output. Overall, focus on both downside and upside risks seem to provide stabilization benefits where VaR and Wang fail.



(a) systemic risk measures

Figure 20: Uncertainty shock with macroprudential KT



(a) systemic risk measures

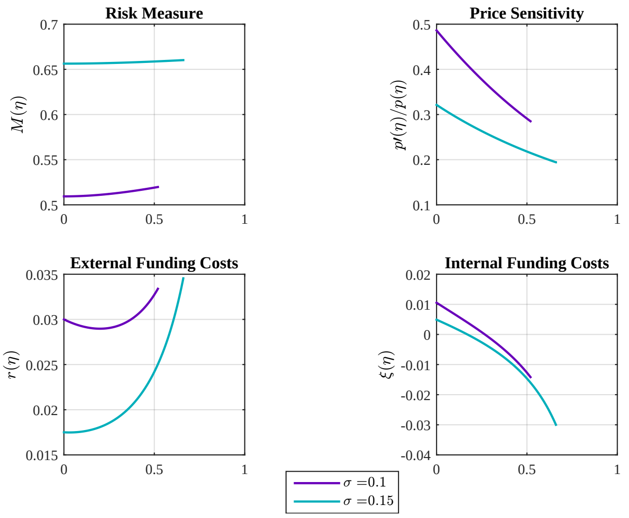
Figure 21: Uncertainty shock with macroprudential anti-KT

Table 2 conveys this result.

To put this logic in perspective, under KT and anti-KT regulation when banks' capital declines natural investors value capital less in booms, and sell it to marginal investors at the price discount.[47] In contrast, when tail losses are salient for regulators natural investors buy capital at price discount.[48] Moreover, the selling price is higher than the buying price. The reason why selling the risky asset depresses its price to a lower degree, is because regulators focus on favorable realizations of the risky asset exactly when natural investors become less optimistic. However, selling or buying of the risky asset increases systemic risk but dampens endogenous

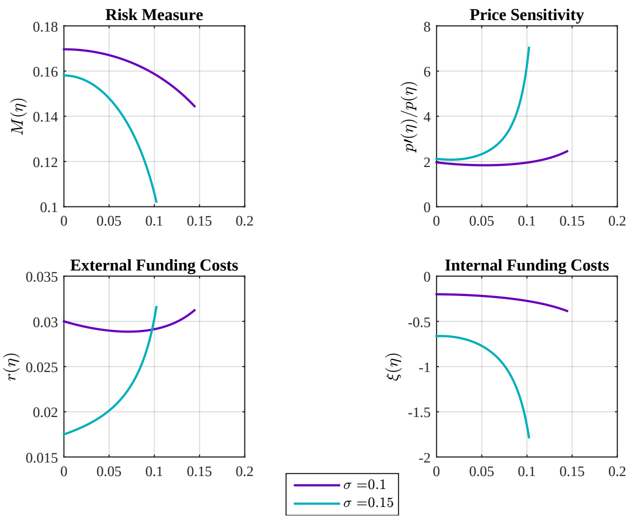
[47] See Table 3 and Table 4.

[48] See Table 1 and Table 5.



(a) risk attitudes

Figure 22: Uncertainty shock with macroprudential Wang

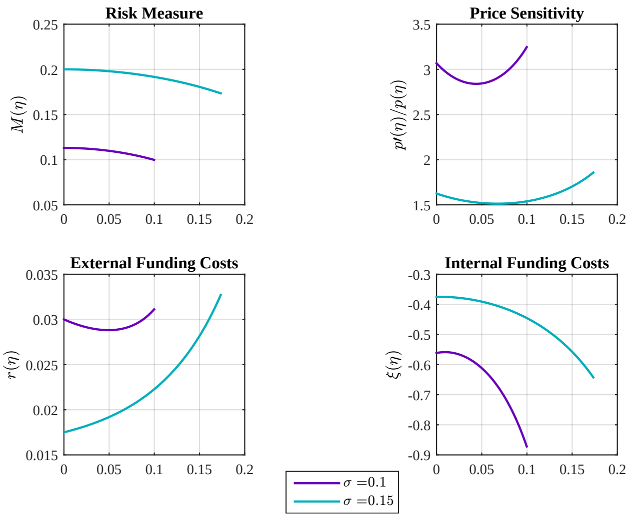


(a) risk attitudes

Figure 23: Uncertainty shock with macroprudential KT

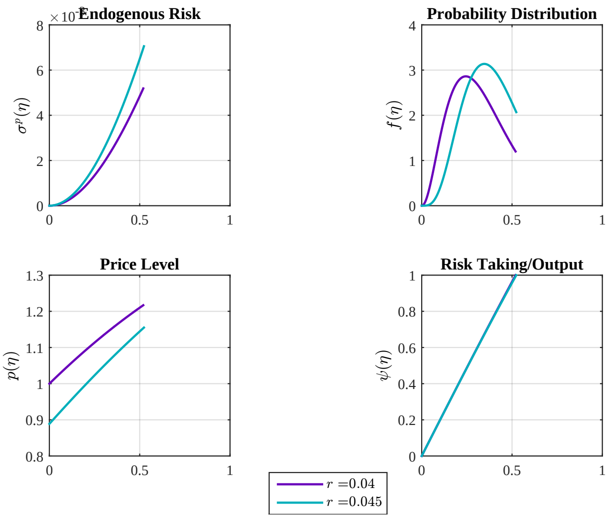
risk. In contrast, in Brunnermeier and Sannikov (2014) selling risky assets leads to amplification because recovery from the undercapitalized region is slow.

The reverse holds when external funding is impaired. When marginal investors suddenly increase borrowing costs and value capital less in busts, under KT and anti-KT regulation intermediaries buy capital while under VaR and Wang they sell it. Once again, the selling price is higher than the buying price. In the event of distressed lenders, the price decline under Wang and VaR is lower because these policies reduce the systemic risk more. Focusing on the downside potential when households become more pessimistic about capital brings a higher accumulation of net worth by raising capital requirements. Also, borrowers become less risk-



(a) risk attitudes

Figure 24: Uncertainty shock with macroprudential anti-KT

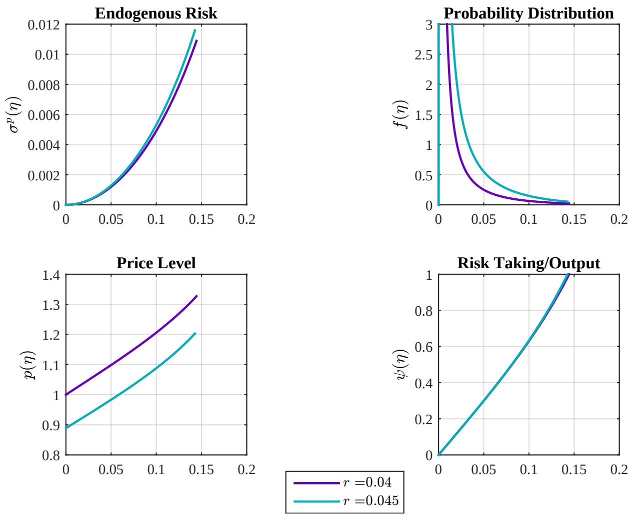


(a) systemic risk measures

Figure 25: Increase in borrowing costs with macroprudential Wang

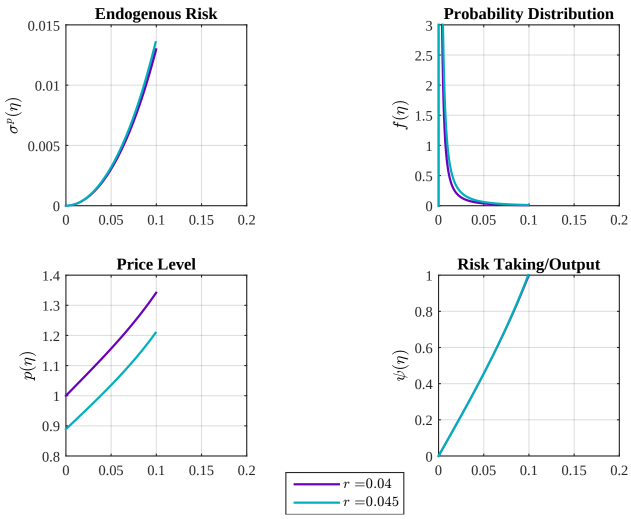
seeking and risk-taking. In contrast, under KT and anti-KT capital buffer drops and intermediaries become more risk-seeking and risk-taking.

Yet some implications and tradeoffs remain independent of which losses are salient for regulators. For instance, all four macroprudential instruments can either reduce endogenous or systemic risk in the face of external and internal funding shocks. Second, risk attitudes of households are stable across different regulations, reflecting foremost their unconstrained choice and attitudes, and their precautionary incentive to save after an uncertainty shock. Finally, banks appear to underestimate potential losses relatively more in booms. Therefore, the procyclicality of risk attitudes seems to be an invariant characteristic of the financial sector.



(a) financial and real variables

Figure 26: Increase in borrowing costs with macroprudential KT

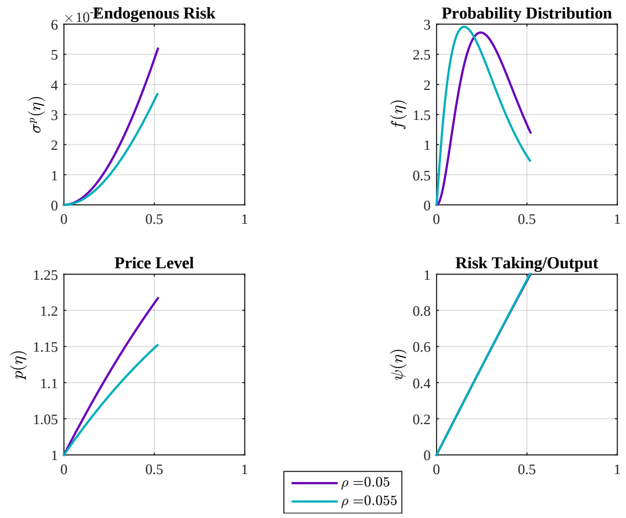


(a) financial and real variables

Figure 27: Increase in borrowing costs with macroprudential anti-KT

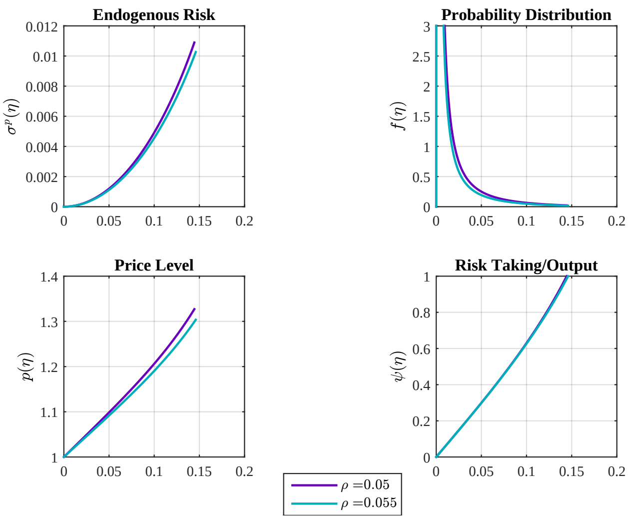
Ideally, policymakers strive for one prudential tool to combat various financial vulnerabilities against different adverse shocks. Our proposed regulation can be seen as a second-best approximation to this ideal, one that can be comfortably achieved by switching between regulatory regimes. We have shown that salience of tail losses is beneficial for policymakers aimed to limit the intermediaries' risk-taking and to reduce the likelihood of systemic crises. Conversely, if intermediate losses are the focus of attention, this helps build a more resilient financial system where systemic risk is unaffected in the presence of market disruptions.

When designing capital requirements, policymakers need to account for the source of the shock and priorities of regulators. Only a financial system that is characterized by low systemic



(a) systemic risk measures

Figure 28: Credit crunch with macroprudential Wang

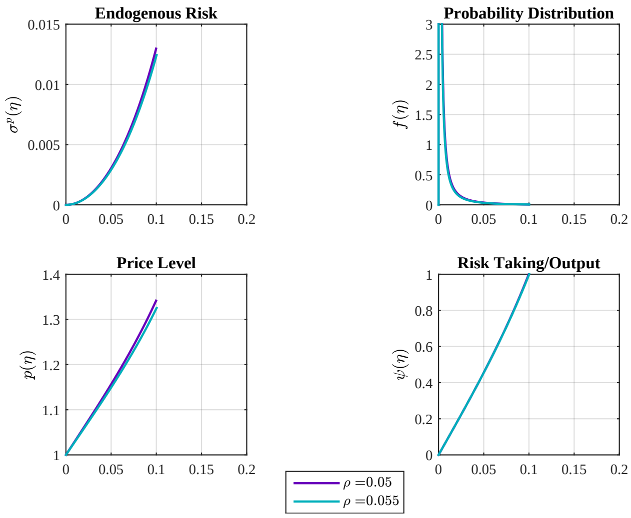


(a) systemic risk measures

Figure 29: Credit crunch with macroprudential KT

risk and resilient to shocks should be the goal of regulators. Capital buffers that adequately weigh ex-ante prevention of systemic risk and ex-post crisis management might be capable of achieving this mission. In our framework, this objective translates into weighing downside risk measures and upside risk measures according to regulators' preferences for risk reduction or resilience. In the Basel III regulatory framework, the objective can be implemented as a weighted average of countercyclical and procyclical capital requirements. The advantage of our model over current Basel III implementation is that capital ratios endogenously emerge.

Alternatively, regulators may enforce *VaR* or Wang policies during peaceful times while adjusting the choice of a risk measure when markets are disrupted, where for instance mitigat-



(a) systemic risk measures

Figure 30: Credit crunch with macroprudential anti-KT

ing systemic risk takes precedence over output loss or vice versa. Nonetheless, we suggest our model provides a parsimonious account for the relationship between systemic risk and macroprudential tools based on salience and probability weighting through underlying shifts in risk attitudes. In this respect, it is an important step towards shifting the frontiers of a positive analysis of new macroprudential frameworks.

5.1 Connection to Salience and Prospect Theory

We take stocks by comparing our predictions to those of prospect and salience theory. Our first result related to two theories is a twofold pattern under VaR and Wang: the financial sector is either risk-averse or risk-seeking in market losses of low probability, while regulators are risk-averse. The second prediction implies that banks are risk-seeking in market losses under KT and anti-KT regulation. Importantly, in our model, the role of the generator of regulators' risk attitudes is taken by the probability weighting function (Figure 16), while risk attitudes of intermediaries endogenously emerge.

In decision making under risk, prospect and salience theories tackle the question of why do agents prefer sometimes to take the risk but sometimes avoid risk? Specifically, both theories yield a fourfold choice pattern in risk preferences: decision-makers are risk-seeking in losses of high probability and risk-averse in losses of low probability, and vice versa risk-seeking in gains of low probability and risk-averse in gains of high probability. To understand it in more detail, prospect theory distinguishes two drivers of risk attitudes: the curvature of the value function (it is concave for gains and convex for losses) and of probability weighting function (the possibility and the certainty effect). The value function captures an observation that agents perceive outcomes as gains and losses from the reference point, and that ”losses loom larger than gains”. Subjective probabilities, on the other hand, reflect the tendency of the individual to pay comparatively more attention to less probable outcomes. While value function and the certainty effects favor risk aversion for gains and risk-seeking for losses, the possibility effect favors risk seeking for gains and risk aversion for losses. Three effects combined produce an observed fourfold pattern. Instead, the salience theory yields the fourfold pattern solely based on the salience of payoffs. The more payoff is different than residual payoffs, the more it is overweighted in decisions. This is an important difference from prospect theory as inverse S-shaped probability weighting function we have seen in Figure 16 is context-dependent. It means that the threshold where individuals overweight or underweight probabilities change with the state of the world.

In prospect theory, individuals tend to shift from risk aversion in losses to risk-seeking if the possibility effect reflects into certainty effect. This occurs when the probability of a loss jumps from small to large. Since with VaR_α the probability of a loss is fixed at level α, we interpret our prediction of a shift in risk attitudes consistent with salience theory. In salience theory, the decision-maker is risk-seeking when a lottery's upside is salient and risk-averse when its downside is salient. Importantly, the preference shift occurs because there is a shift in salience from downside to upside, where the upside is salient when gains are larger than losses. The second prediction of why banks become risk-seeking under KT and anti-KT regulation can be reconciled with both theories. One explanation is that the aforementioned risk-management policies may elicit risk-seeking preferences because both measures draw attention to the upside potential of the risky asset, which is consistent with salience theory. Alternatively, intermediaries hope to avoid market risk when the mere possibility of market loss becoming a certainty, which is in line with prospect theory.

Our model also predicts conditions under which preference reversal described by Lichtenstein and Slovic (1971) occur. The key idea of this phenomenon is that preferences, as revealed by choice, are the opposite of preferences as revealed by pricing. For example, the decision-maker exhibiting preference reversal may choose a safe bet(with a large probability of a small gain) over a long shot (with a small probability of a larger gain). However, when buying or selling each lottery, they state a higher minimum buying or selling price for the long-shot lottery. In other words, the decision-maker is risk-averse in choice but risk-seeking in pricing. The reverse may also hold, and the decision maker's behavior can evince risk-seeking choice and risk-averse pricing. This divergence of pricing and choice occurs twice in our model. In the first case, a decline in bank capital elicits preference reversal of banks when regulators focus on tail losses with VaR or Wang.[49] In the second case, reversal is evoked by tightening of external funding under KT and anti-KT regulation.[50] In both cases, the financial sector increases risky asset holdings by buying additional capital but is willing to pay less for the riskier asset. The salience theory predicts that preference reversal between choice and pricing is a consequence of lottery evaluation in different contexts that affect salience. For example, if the decision-maker compares two assets in a choice context he prefers the safer one, while in isolation context his valuation of the risky asset is higher. Since prospect theory of Tversky and Kahneman (1992)

[49]See Table 1 and 5, column "ρ", rows "Fire-sale discount" and "Output".

[50]See Table 3 and 4, column "r", rows "Fire-sale discount" and "Output".

cannot generate preference reversal, we interpret our result in line with salience theory.

6 Conclusion

The role of macroprudential policy instruments to mitigate the probability and the severity of systemic crises have received substantial attention in the great recession aftermath. In this paper, we incorporate capital requirements in the form of four market risk measures into a continuous-time heterogeneous agent model. The key idea of proposed spectral risk measures is that regulators overweight market losses that drive their attention. We find that salience of small probability market losses induce risk-averse preferences of the financial sector when the risky asset upside is salient and risk-seeking preferences when the downside is salient. This occurs with *VaR* and Wang's capital requirements, which are countercyclical. In contrast, when regulators also focus on the upside potential of the risky asset with KT and anti-KT, the financial sector exhibits risk-seeking preferences, and capital requirements are procyclical.

Given four regulatory frameworks, we examine the efficiency of macroprudential instruments in response to adverse shocks such as a sudden increase in borrowing costs, a credit crunch or an increase in uncertainty. Crucially, we argue that regulators should complementary use proposed market risk measures to help reduce key threats to financial stability. Focusing exclusively on the market downside with *VaR* and Wang is beneficial for policymakers aimed to limit the intermediaries' risk-taking and reduce the likelihood of systemic crises. On the other hand, focusing on both the market downside and upside helps build a more resilient financial system. The results suggest that *VaR* and Wang's risk measure are effective in controlling endogenous risk and the likelihood of systemic crisis in environments prone to uncertainty shocks, but unable to prevent output decline and fire sales. By comparison, with KT risk measure output increases in the face of uncertainty shock. In managing adverse inside or outside liquidity shocks, all four measures can either reduce the ex-post crisis anticipation or amplification, but cannot control both. We also find that benefits of longer-term or longer-shot *VaR* regulation include alleviating systemic and endogenous risk, however, entail costs in terms of lower output.

In future research, we would like to develop a richer theoretical model that allows for monetary and macroprudential policy interaction, and design a more sophisticated prudential policy instruments under uncertainty, such as backward-looking risk measures where past market conditions are relevant for future risk assessment or the robust *VaR* and spectral risk measures, which are applicable in situations where only partial information on the underlying probability distribution is given. The second direction is of great practical value in accounting for more pragmatic aspects of policy implementation. Finally, a missing and the most promising element in the future analysis is the empirical estimation of the weighting function. This estimation requires an episode when the financial sector was undercapitalized and the capital requirement constraint was binding. A way to move in this direction would be to estimate the financial sector's asset pricing equation with Generalized Method of Moments from the data of banks that were recapitalized during the recent crisis.

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A Omitted Proofs and Definitions

In this appendix, we provide mathematical definitions of coherent risk measures and quantiles. We also provide proofs of propositions stated in Section 3.

A risk measure is defined by the following four coherency axioms Artzner et al. (1999). For both definitions, let X be a random variable that represents future uncertain profit or loss.

Definition 1 (Risk measure) Let V be a set of all real-valued random variables on a probability space (Ω, Σ, P). A function $\rho : V \to \mathbb{R}$ is called a coherent risk measure if it satisfies following axioms:

1. (Translation invariance) For all $X \in V$ and all $a \in \mathbb{R}$, $\rho(X + a) = \rho(X) - a$.
2. (Subadditivity) For all X and $Y \in V$, $\rho(X + Y) \leq \rho(X) + \rho(Y)$.
3. (Positive homogeneity) For all $h \geq 0$, $\rho(hX) = h\rho(X)$.
4. (Monotonicity) For all X and $Y \in V$, with $X \leq Y$, we have $\rho(Y) \leq \rho(X)$.

Definition 2 (Quantiles) Lower α-quantile of a random variable X is defined by $q_\alpha(X) = \inf\{x \in \mathbb{R} : \quad P[X \leq x] \geq \alpha\}$, $\alpha \in (0, 1)$.

Proof of Proposition 1. Recall that we defined market losses as $X_t - X_{t+\tau}$ and $VaR_\alpha^{t,t+\tau}$ is the quantile with confidence level $1 - \alpha$ of market losses. In other words, $VaR_\alpha^{t,t+\tau}$ is defined as

$$VaR_\alpha^{t,t+\tau} = \inf\{L \geq 0 : P(X_t - X_{t+\tau} \geq L|\mathcal{F}_t) \leq \alpha\} = (Q_{t,t+\tau}^\alpha)^-,$$

where

$$Q_{t,t+\tau}^\alpha = \sup\{L \in \mathbb{R} : P(X_{t+\tau} - X_t \leq L|\mathcal{F}_t) \leq \alpha\}$$

is the quantile of the projected market gains over the horizon of length τ and $x^- = \max\{0, -x\}$. Then we have

$$\begin{aligned}
&P(X_{t+\tau} - X_t \leq L|\mathcal{F}_t) \\
&= P\left(X_t \exp\left(\int_t^{t+\tau} (\mu_s^p + \sigma\sigma_s^p - \frac{1}{2}(\sigma + \sigma_s^p)^2)ds + \int_t^{t+\tau} (\sigma + \sigma_s^p)dW_s\right) - X_t \leq L \quad |\mathcal{F}_t\right) \\
&= P\left(\exp\left((\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma + \sigma_t^p)^2)\tau + (\sigma + \sigma_t^p)(W_{t+\tau} - W_t)\right) \leq 1 + \frac{L}{X_t} \quad |\mathcal{F}_t\right) \\
&= P\left((\sigma + \sigma_t^p)(W_{t+\tau} - W_t) \leq \log\left(1 + \frac{L}{X_t}\right) - (\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma + \sigma_t^p)^2)\tau \quad |\mathcal{F}_t\right) \\
&= \Phi\left(\frac{\log\left(1 + \frac{L}{X_t}\right) - (\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma + \sigma_t^p)^2)\tau}{(\sigma + \sigma_t^p)\sqrt{\tau}}\right)
\end{aligned}$$

where the last equality follows from the fact that the random variable $(\sigma + \sigma_t^p)(W_{t+\tau} - W_t)$ is conditionally normally distributed with zero mean and variance $(\sigma + \sigma_t^p)^2\tau$, and $\Phi(\cdot)$ is the cumulative distribution of the standard normal distribution. Therefore, we have

$$
\begin{aligned}
&P(X_{t+\tau} - X_t \leq L|\mathcal{F}_t) \leq \alpha \\
&\Phi\left(\frac{\log\left(1+\dfrac{L}{X_t}\right) - (\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau}{(\sigma+\sigma_t^p)\sqrt{\tau}}\right) \leq \alpha \\
&L \leq X_t\left(\exp\left((\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau + \Phi^{-1}(\alpha)(\sigma+\sigma_t^p)\sqrt{\tau}\right) - 1\right)
\end{aligned}
$$

which implies

$$Q^{\alpha}_{t,t+\tau} = X_t\left(\exp\left((\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau + \Phi^{-1}(\alpha)(\sigma+\sigma_t^p)\sqrt{\tau}\right) - 1\right)$$

Finally, we obtain the expression which is stated in the proposition

$$VaR^{t,t+\tau}_{\alpha} = p_t k_t\left(1 - \exp\left((\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau + \Phi^{-1}(\alpha)(\sigma+\sigma_t^p)\sqrt{\tau}\right)\right).$$

□

PROOF OF PROPOSITION 2. The proof for the optimal policy functions of households' we give by the method of matching drifts, and the proof for the value function by guess and verify method. For the ease of exposition we omit the time subscript.

Step 1

We define by $\lambda = V'(\underline{n})$ the marginal utility of net worth or the stochastic discount factor of households, which follows the Brownian motion

$$d\lambda = \mu^{\lambda}\lambda dt + \sigma^{\lambda}\lambda dW_t.$$

By Ito's lemma we have

$$\mu^{\lambda}\lambda = V''(\underline{n})[a\underline{k} + r_t\underline{n} + p\underline{k}(\mu^p + \sigma\sigma^p - r_t) - c] + \frac{1}{2}V'''(\underline{n})(\sigma+\sigma^p)^2 p^2 \underline{k}^2$$

$$\sigma^{\lambda}\lambda = V''(\underline{n})(\sigma+\sigma^p)p\underline{k}.$$

Step 2

We obtain the envelope condition after substituting for households' FOCs from the main text

$$(r - r_t)V'(\underline{n}) = V''(\underline{n})[a\underline{k} + r_t\underline{n} + p\underline{k}(\mu^p + \sigma\sigma^p - r_t) - c] + \frac{1}{2}V'''(\underline{n})(\sigma+\sigma^p)^2 p^2 \underline{k}^2$$

which implies

$$(r - r_t)\lambda = \mu^{\lambda}\lambda$$

$$\frac{d\lambda}{\lambda} = (r - r_t)dt + \sigma^{\lambda}dW_t.$$

Step 3

From the first order condition for consumption and log utility, we have that consumption is equal to $c = 1/\lambda$. Using again Ito's lemma for consumption as a function of a stochastic discount factor, we obtain expressions for consumption drift and volatility

$$\mu^c c = -\frac{1}{\lambda^2}\mu^\lambda + \frac{1}{2}\frac{2}{\lambda^3}(\sigma^\lambda)^2\lambda^2, \quad \sigma^c c = -\frac{1}{\lambda^2}\sigma^\lambda \lambda$$

$$\mu^c = -\mu^\lambda + (\sigma^\lambda)^2, \quad \sigma^c = -\sigma^\lambda$$

$$\mu^c = r_t - r + (\sigma^c)^2. \tag{41}$$

Note that we can rewrite households' asset pricing equation as

$$-\sigma^\lambda = \frac{\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t}{\sigma + \sigma^p}. \tag{42}$$

Step 4

We also know by Ito's lemma expressions for consumption drift and volatility as a function of net worth

$$\mu^c c = c'(\underline{n})[\underline{ak} + r_t\underline{n} + p\underline{k}(\mu^p + \sigma\sigma^p - r_t) - c] + \frac{1}{2}c''(\underline{n})(\underline{n})(\sigma + \sigma^p)^2 p^2 \underline{k}^2, \quad \sigma^c c = c'(\underline{n})(\sigma + \sigma^p)p\underline{k}. \tag{43}$$

Using $\sigma^c = -\sigma^\lambda$ and substituting for σ^c in the asset pricing equation, we get

$$\underline{k}(\underline{n}) = \frac{\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t}{(\sigma + \sigma^p)^2}\frac{c(\underline{n})}{c'(\underline{n})p} \tag{44}$$

The function $c(\underline{n})$ can be found by equating drifts in (41) and (43) and using (42) and substituting for k from (44)

$$r_t - r + \left(\frac{\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t}{\sigma + \sigma^p}\right)^2 =$$

$$= \frac{c'(\underline{n})[\underline{ak} + r_t\underline{n} + p\underline{k}(\mu^p + \sigma\sigma^p - r_t) - c(\underline{n})] + \frac{1}{2}c''(\underline{n})(\underline{n})(\sigma + \sigma^p)^2 p^2 \underline{k}^2}{c(\underline{n})}$$

which gives us the optimal policy rule for consumption $c(\underline{n}) = rn$. Then, the capital rule $k(\underline{n})$ is easily obtained by substituting for $c(n) = rn$ and $c'(n) = r$ in (44).

Step 5

Plugging back two policy rules into HJB equation from the main text we get the new HJB equation

$$rV(\underline{n}) = \log(r\underline{n}) + V'(\underline{n})\underline{n}\left[\left(\frac{\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t}{\sigma + \sigma^p}\right)^2 + r_t - r\right] + \frac{1}{2}V''(\underline{n})\underline{n}^2\left(\frac{\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t}{\sigma + \sigma^p}\right)^2.$$

Finally, we guess and verify the value function form to be $V(\underline{n}) = B\log\underline{n} + D$. This functional form implies that $V'(\underline{n}) = B\frac{1}{n}$ and $V''(\underline{n}) = -B\frac{1}{n^2}$. Plugging this back in the new HJB, we find the coefficient to be equal to

$B = \frac{1}{r}$, and $D = \log\frac{1}{r} + \frac{1}{r^2}(r_t - r + \frac{(\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t)^2}{2(\sigma + \sigma^p)^2})$, which concludes the proof. □

PROOF OF PROPOSITION 3. We solve for expert's optimal consumption and capital rules and the value function by using the same methods and steps as in the household's case.

Step 1

Let $\lambda = V'(n)$ represent expert's stochastic discount factor and let it follow Brownian motion

$$d\lambda = \mu^\lambda\lambda dt + \sigma^\lambda\lambda dW_t$$

By Ito's lemma we have

$$\mu^\lambda\lambda = V''(n)[ak + r_t n + pk(\mu^p + \sigma\sigma^p - r_t) - c] + \frac{1}{2}V'''(n)(\sigma + \sigma^p)^2 p^2 k^2$$

$$\sigma^\lambda\lambda = V''(n)(\sigma + \sigma^p)pk$$

Step 2

The envelope condition of experts is

$$\begin{aligned}
\rho V'(n) &= \left(\log c - V'(n)c\right)' + \left(V'(n)\left[a\frac{n}{pM} + r_t n + \frac{n}{M}(\mu^p + \sigma\sigma^p - r_t)\right]\right)' \\
&\quad + \frac{1}{2}\left[V''(n)(\sigma + \sigma^p)^2\frac{n^2}{M^2}\right]' + \xi'(n)\left[n - pkM\right] + \xi(n)\left[n - pMk(n)\right]' \\
&= \left(\frac{1}{c}c'(n) - V'(n)c(n) - V''(n)c\right) + V''(n)\left[a\frac{n}{pM} + r_t n + \frac{n}{M}(\mu^p + \sigma\sigma^p - r_t)\right] \\
&\quad + V'(n)\left[r_t + \frac{1}{M}(a + \mu^p + \sigma\sigma^p - r_t)\right] + V''(n)\left[(\sigma + \sigma^p)^2\frac{n}{M^2}\right] \\
&\quad + \frac{1}{2}\left[V'''(n)(\sigma + \sigma^p)^2\frac{n^2}{M^2}\right]
\end{aligned}$$

The equality follows from the fact that the constraint is binding and when substituting for $k = \frac{n}{pM}$. Using the first order condition for consumption and expression for drift and volatility of the stochastic discount function we get the rewritten envelope condition

$$\rho - \frac{1}{M}(\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t) - r_t = \mu^\lambda + (\sigma + \sigma^p)\frac{\sigma^\lambda}{M}.$$

Therefore, the SDF of experts evolves as

$$\frac{d\lambda}{\lambda} = \left(\rho - r_t - \frac{1}{M}(\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t) - (\sigma + \sigma^p)\frac{\sigma^\lambda}{M}\right) dt + \sigma^\lambda dW_t$$

giving us the expression for the drift of the SDF

$$\mu^\lambda = \rho - r_t - \frac{1}{M}(\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t) - (\sigma + \sigma^p)\frac{\sigma^\lambda}{M}. \tag{45}$$

We can also rewrite the first order condition for capital presented in the main text, that is the expert's asset pricing equation

$$\lambda\left(a + p(\mu^p + \sigma\sigma^p - r_t)\right) + (\sigma + \sigma^p)\sigma^\lambda\lambda p - \xi p M = 0. \tag{46}$$

Then we get that expert's SDF which differs from household's SDF exactly in the third term

$$\mu^\lambda = \rho - r_t - \frac{\xi}{\lambda}.$$

If experts were unconstrained, ξ would be equal to zero and we would have the unconstrained financial sector without regulators imposing the capital requirement constraint, as in Brunnermeier and Sannikov (2014). Here, ξ could be interpreted as a marginal cost of the financial regulation in terms of a unit of net worth.

Step 3

The first order condition with respect to consumption is the same as in the household's case, $c = \frac{1}{\lambda}$. Using Ito's lemma we obtain the same expressions for consumption growth and volatility as in the household's case

$$\mu^c = -\mu^\lambda + (\sigma^\lambda)^2, \quad \sigma^c = -\sigma^\lambda. \tag{47}$$

Step 4

We also know by Ito's lemma $c(n)$

$$\mu^c c = c'(n)[ak + r_t n + pk(\mu^p + \sigma\sigma^p - r_t) - c] + \frac{1}{2}c''(n)(\sigma + \sigma^p)^2 p^2 k^2, \tag{48}$$

$$\sigma^c c = c'(n)(\sigma + \sigma^p)pk = c'(n)(\sigma + \sigma^p)\frac{n}{M}. \tag{49}$$

Performing the same steps as in the households case, by matching consumption drifts using

(48),(49),(47),(46), and (45) we get

$$r_t - \rho + \frac{1}{M}(\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t) + \frac{(\sigma+\sigma^p)}{M}\left(-\frac{c'(n)(\sigma+\sigma^p)\frac{n}{M}}{c(n)}\right) + \left(-\frac{c'(n)(\sigma+\sigma^p)\frac{n}{M}}{c(n)}\right)^2$$
$$= \frac{c'(n)[\frac{n}{M}(\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t) + r_t n - c(n)] + \frac{1}{2}c''(n)(\sigma+\sigma^p)^2\frac{n^2}{M^2}}{c(n)}.$$

Guessing a linear consumption rule we get $c(n) = An + F$ and substituting it in matching drifts we get $c(n) = \rho n$.

Step 5

Plugging back policy rules into expert's HJB equation, we get coefficients

$$\rho V(n) = \log(\rho n) + V'(n)n\left[\frac{1}{M}(\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t) + r_t - \rho\right] + \frac{1}{2}V''(n)\frac{n^2}{M^2}(\sigma+\sigma^p)^2.$$

We guess and verify the value function form $V(n) = B\log n + D$. Plugging this back in HJB we get $B = \frac{1}{\rho}$ and $D = \log\frac{1}{\rho} + \frac{1}{\rho^2}\left(r_t - \rho + \frac{1}{M}(\frac{a}{p} + \mu^p + \sigma\sigma^p - r_t) - \frac{1}{2}\frac{(\sigma+\sigma^p)^2}{M^2}\right)$, which concludes the proof.

□

Proof of Proposition 4. Recall that experts' net worth evolves as

$$\frac{dn_t}{n_t} = r_t dt + \frac{p_t k_t}{n_t}(\frac{a}{p_t} + \mu_t^p + \sigma\sigma_t^p - r_t)d_t - \frac{c_t}{n_t}d_t + \frac{p_t k_t}{n_t}(\sigma + \sigma_t^p)dW_t.$$

We know from Proposition 2 that expert's optimal capital and consumption policy functions are $k_t = \dfrac{n_t}{p_t M_t}$ and $c_t = \rho n_t$, respectively, which gives us

$$\frac{dn_t}{n_t} = \left(r_t + \frac{1}{M_t}(\frac{a}{p_t} + \mu_t^p + \sigma\sigma_t^p - r_t) - \rho\right)d_t + \frac{1}{M_t}(\sigma + \sigma_t^p)dW_t.$$

Further, total capital in the economy evolves as

$$\frac{d(p_t K_t)}{p_t K_t} = (\mu_t^p + \sigma\sigma_t^p)d_t + (\sigma + \sigma_t^p)dW_t.$$

Using Ito's formula for law motion of ratio of two geometric Brownian motions [51] where $\eta_t =$

[51] If we have two GBMs $\dfrac{dX_t}{X_t} = \mu_t^x d_t + \sigma_t^x dW_t$ and $\dfrac{dY_t}{Y_t} = \mu_t^y d_t + \sigma_t^y dW_t$, then we have
$\dfrac{dX_t/Y_t}{X_t/Y_t} = (\mu_t^x - \mu_t^y + (\sigma_t^y)^2 - \sigma_t^x\sigma_t^y)d_t + (\sigma_t^x - \sigma_t^y)dW_t$.

$\frac{n_t}{p_t K_t}$, we obtain

$$
\begin{aligned}
\frac{d\eta_t}{\eta_t} &= \left(r_t + \frac{1}{M_t}(\frac{a}{p_t} + \mu_t^p + \sigma\sigma_t^p - r_t) - \rho - \mu_t^p - \sigma\sigma_t^p + (\sigma + \sigma_t^p)^2 - \frac{1}{M_t}(\sigma + \sigma_t^p)^2 \right) dt \\
&+ (\frac{1}{M_t} - 1)(\sigma + \sigma_t^p) dW_t \\
&= \left(\frac{1}{M_t}\frac{a}{p_t} - \rho + (\frac{1}{M_t} - 1)(\mu_t^p + \sigma\sigma_t^p - r_t - (\sigma + \sigma^p)^2) \right) d_t + (\frac{1}{M_t} - 1)(\sigma + \sigma_t^p) dW_t.
\end{aligned}
$$

□

Proposition 6. *The stationary wealth distribution $f(\eta_t)$ satisfies Kolmogorov forward equation*

$$0 = -\frac{\partial}{\partial\eta}(\mu_t^\eta(\eta)\eta_t f(\eta_t)) + \frac{1}{2}\frac{\partial^2}{\partial\eta^2}((\sigma_t^\eta(\eta_t)\eta_t)^2 f(\eta_t)) \tag{50}$$

on a closed interval. We assume that detailed balance condition holds, meaning that no probability can "escape" from the interval $[0, \eta^*]$. [52] *In particular, we assume that η_t is equipped with reflecting boundary conditions, one at $\eta = 0$ and the endogenous reflecting boundary $\eta = \eta^*$. Then the stationary distribution is equal to $f(\eta) = C\frac{e^{2\int_0^\eta \frac{\mu^\eta(\eta')}{\sigma^\eta(\eta')^2\eta'}d\eta'}}{\sigma^\eta(\eta)^2\eta^2}$.*

Proof of Proposition 6. The stationary Kolmogorov forward equation (50) can be rewritten as

$$\frac{dJ}{d\eta} = 0,$$

where $J(\eta) = -\mu_t^\eta(\eta)\eta + \frac{1}{2}\frac{\partial}{\partial\eta}((\sigma_t^\eta(\eta)\eta)^2 f(\eta))$ denotes the probability flux or the probability current associated with the equation (50). Since the derivative of the probability current is equal to zero for all $\eta \in [0, \eta^*]$, this means that the current is constant at the same interval, that is

$$J(\eta) = const.$$

From the reflecting boundary conditions we have $J(0) = J(\eta^*) = 0$, and from the integration of Kolmogorov forward equation we conclude that probability current must be constant and equal to 0 since $J(0) = 0$. Consequently, we have $J(\eta) = 0$ for $\eta \in [0, \eta^*]$. Hence, the stationary

[52] This is a sufficient condition for the stationary wealth share distribution to exist. Detailed balance condition or reversibility property of Markov chain is a sufficient but not a necessary condition in order to have a stationary distribution. Other possible type of boundary condition is absorbing, $f(0) = 0$, implying that some probability mass can leave the domain $[0, \eta^*]$. Intuitive interpretation can be explained as follows. Suppose we have two state Markov chain with two states of the world, a good state when experts have enough capital and a bad state when experts' capital is scarce $\{g, b\}$. Let π_g and π_b denote the probability of being in a good and bad state respectively (mass of experts who are sufficiently and insufficiently capitalized) and T_{gb} and T_{bg} transition probabilities from a good to a bad state and vice versa from a bad to a good state. Reversibility reads $\pi_g T_{gb} = \pi_b T_{bg}$, a mass of experts moving from sufficient to insufficient capital is equal to a mass of experts moving from insufficient to sufficient capital, probability inflow to a good state is equal to probability outflow from a good state, both states are visited in equilibrium. Absorbing conditions would imply that in equilibrium we end up in a good or a bad state, i.e. probability distribution is degenerate. Since in our case η is a continuous state variable, detailed balance implies no discontinuities in probability and its first derivative.

Kolmogorov equation becomes

$$0 = -\mu_t^{\eta}(\eta)\eta + \frac{1}{2}\frac{\partial}{\partial\eta}((\sigma_t^{\eta}(\eta)\eta)^2 f(\eta)) \tag{51}$$

Integrating the equation (51), we obtain the closed form solution in the main text, where C is the normalization constant.[53] □

PROOF OF M_{KT}. We need to evaluate the following integral

$$\begin{aligned}
M_{KT} &= \int_0^1 (ap^2 + bp^2 + c)(1 - e^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau + \Phi^{-1}(p)(\sigma+\sigma_t^p)\sqrt{\tau}})dp \\
&= \underbrace{\int_0^1 (ap^2 + bp^2 + c)dp}_{=1} - \int_0^1 (ap^2 + bp^2 + c)e^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau + \Phi^{-1}(p)(\sigma+\sigma_t^p)\sqrt{\tau}} dp \\
&= 1 - (I + II + III)
\end{aligned}$$

which can be separated into the sum of three integrals. Let us first introduce the change of variables $\Phi^{-1}(p) = x$, $p = \Phi(x)$, $dp = \phi(x)dx$ in order to solve these integrals. The third integral is equal to

$$\begin{aligned}
III &= ce^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau} \int_{-\infty}^{\infty} e^{(\sigma+\sigma_t^p)\sqrt{\tau}x}\phi(x)dx \\
&= ce^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2\tau + \frac{1}{2}(\sigma+\sigma_t^p)^2\tau}\Phi(x - (\sigma + \sigma^p)\sqrt{\tau})\Big|_{-\infty}^{+\infty} \\
&= ce^{(\mu_t^p + \sigma\sigma_t^p)\tau}
\end{aligned}$$

, where we have used $\int e^{nx}\phi(x)dx = e^{\frac{n^2}{2}}\Phi(x-n)$[54] . The second integral is equal to

$$II = be^{(\mu_t^p + \sigma\sigma_t^p - \frac{1}{2}(\sigma+\sigma_t^p)^2)\tau} \int_{-\infty}^{\infty} e^{(\sigma+\sigma_t^p)\sqrt{\tau}}\phi(x)\Phi(x)dx.$$

Introducing the integration by parts $u = \Phi(x)$ $du = \phi(x)dx$, and $dv = e^{(\sigma+\sigma_t^p)\sqrt{\tau}x}\phi(x)dx$,

[53]If the boundary conditions are absorbing then the diffusion process η_t would eventually be absorbed by the boundary points 0 and η^*. Consequently, the stationary distribution is 0 (degenerate). If the boundary conditions were periodic, this would imply $C_1 = -\mu_t^{\eta}(\eta)\eta + \frac{1}{2}\frac{\partial}{\partial\eta}((\sigma_t^{\eta}(\eta)\eta)^2 f(\eta))$ where the constant C_1 is determined from the periodic boundary conditions $f(0) = f(\eta^*), J(0) = J(\eta^*)$. Periodic boundary conditions are used to model small-system processes that are part of a large system that exhibits fixed periodicity. For instance, if the periodicity of systemic crises is fixed, and financial sector constitutes a small part of an economy. This conditions could be possibly used in a large scale continuous time DSGE models.

[54]See wikipedia: List of integrals of Gaussian functions.

$v = e^{(\sigma+\sigma_t^p)^2\tau/2}\Phi(x-(\sigma+\sigma^p)\sqrt{\tau})$, we have

$$II = be^{(\mu_t^p+\sigma\sigma_t^p-\frac{1}{2}(\sigma+\sigma_t^p)^2)\tau+\frac{1}{2}(\sigma+\sigma_t^p)^2)\tau}\left(\Phi(x)\Phi(x-(\sigma+\sigma^p)\tau)\Big|_{-\infty}^{+\infty} - \int_{-\infty}^{\infty}\phi(x)\Phi(x-(\sigma+\sigma^p)\sqrt{\tau})dx\right)$$
$$= be^{(\mu_t^p+\sigma\sigma_t^p)\tau}(1-1+\Phi(\frac{(\sigma+\sigma^p)\sqrt{\tau}}{\sqrt{2}})) = e^{(\mu_t^p+\sigma\sigma_t^p)\tau}\Phi(\frac{(\sigma+\sigma^p)\sqrt{\tau}}{\sqrt{2}}),$$

where we have used

$$\int_{-\infty}^{+\infty}\Phi(x)\cdot\phi(m+sx)dx = \int_{-\infty}^{+\infty}\Phi(\frac{x-m}{s})\cdot\phi(x)\frac{dx}{s}$$
$$= \frac{1}{s}\left(1-\Phi(\frac{m}{\sqrt{1+s^2}})\right).$$

Analogously, using the integration by parts $u = \Phi^2(x)$ $du = 2\phi(x)\Phi(x)dx$, and $dv = e^{(\sigma+\sigma_t^p)\sqrt{\tau}x}\phi(x)dx$, $v = e^{(\sigma+\sigma_t^p)^2\tau/2}\Phi(x-(\sigma+\sigma^p)\sqrt{\tau})$ and using

$$\int_{-\infty}^{+\infty}\Phi(x)^2\cdot\phi(m+sx)dx = \int_{-\infty}^{+\infty}\Phi^2(\frac{x-m}{s})\cdot\phi(x)\frac{dx}{s}$$
$$= \frac{1}{s}\left(1-\Phi(\frac{m}{\sqrt{1+s^2}})\right) - \frac{2}{s}T\left(\frac{m}{\sqrt{1+s^2}}, \frac{s}{\sqrt{2+s^2}}\right),$$

we find the third integral to be equal to

$$III = ae^{(\mu_t^p+\sigma\sigma_t^p)\tau}\left(\Phi(\frac{(\sigma+\sigma^p)\sqrt{\tau}}{\sqrt{2}}) - T(\frac{(\sigma+\sigma^p)\sqrt{\tau}}{\sqrt{2}}, \frac{1}{\sqrt{3}})\right),$$

which concludes the proof. □

B Transmission Channels

B.1 Balance Sheet Channel and Macroprudential Regulation

In models without borrowing frictions, any investment with positive net present value is undertaken. Modigliani-Miller theorem holds and capital structure is irrelevant for the firm's investment decisions. With borrowing constraint, the choice of external and internal funding is relevant. Seminal contributions of Bernanke et al. (1999) and Kiyotaki and Moore (1997) imply that with costly verifiable project outcome or imposed collateral constraint, the firm's investment and debt level are increasing in internal funds. Therefore, borrowers with higher current cash flow or net worth receive larger financing from creditors. Entrepreneurs need "skin in the game".

In the economy with well-functioning credit markets, only more productive experts would run businesses while unproductive households would lend their money to experts. Reallocating capital to borrowers with the higher marginal product would increase the country's GDP. Failure to reallocate capital to experts due to borrowing frictions is known as a capital *misallocation* problem and shows up empirically as low total factor productivity. This notion of misallocation focuses only on benefits in terms of productivity gains but neglects the costs in terms of higher market risk arising when experts hold a higher share of capital in the economy. A possible goal of welfare maximizing risk measure might weight productivity gains against increased market risk. The net worth channel in investment is represented here by experts' capital share $\psi(\eta) = \frac{\eta}{M(\eta)}$, and as in preceding literature investment is increasing in net worth, albeit declining in the future possible losses evaluated by the regulatory risk measure. The immediate inference we can draw is that if capital requirements increase in a one-to-one fashion with experts wealth share there is no misallocation in a classical sense. Macroprudential regulation alters experts' costs and benefits of holding risky capital by limiting leverage, which depends on the salience of unexpected market losses. With the marked-to-market asset side of the balance sheet, experts internalize market upside and downside risk; internal funds absorb losses suffered from capital investments. Therefore, price variations lead to net worth variations.[55]

B.2 Fire Sales, Asset Price Feedback and Risk Measure

During the recent financial crisis, we witnessed simultaneous housing price decline and fire sales of mortgage-backed securities, which further exacerbated distress. The fire-sale mechanism in Kiyotaki and Moore (1997) and Brunnermeier and Sannikov (2014) work as follows. Consider an indebted expert with insufficient current cash flow to meet his interest obligation. Expert's inability to reschedule debt or borrow more forces him to liquidate assets to repay the debt to creditors. Potential buyers are marginal investors(unproductive households) who value capital less or natural investors(productive experts). With aggregate shocks, all experts are simultaneously distressed, and capital is liquidated at a fire-sale discount to marginal investors. Whenever natural owners of an asset sell it to low valuation users with a downward sloping demand, we expect to observe low equilibrium asset prices. The more capital experts are forced to sell to households, the higher is the price discount. This is the standard notion of fire sales, which can also involve a negative volatility spiral in which lower equilibrium prices become more volatile, which further lowers equilibrium prices, and so on. We refer to this sensitivity of price to experts' net worth as the asset price feedback, which affects the expected growth and volatility of prices.[56] In Brunnermeier and Sannikov (2014), fire sales lead to an increase in price volatility and a rise in the precautionary motive(risk premium), which further depresses

[55] In a run-up to the recent financial crisis, securitization de facto resulted in the marked-to-market balance sheet of banks, since the major share of banks assets such as housing loans were repackaged according to credit default risk and sold as mortgage-backed securities at the market. Equation (29) depicts the expected growth of net worth while equation (28) describes the volatility of the expected net worth growth.

[56] See equations (31), (28) and (30). For the fire-sale mechanism to work, two conditions are necessary. If capital is specialized, this translates into the price impact of the second best use of capital. Second, asset prices should respond to wealth, in particular, the net worth of experts. In this respect, in a representative agent model with the same productivity and impatience levels fire sales do not emerge. We can see this from equation (31). If $r = \rho$ and $a = \underline{a}$, the price is equal to $\frac{a}{\rho}$. Moreover, the price is insensitive to changes in experts' wealth share in the representative agent model, i.e. $p'(\eta) = 0$. From equation (28) and (30) we can see that the expected asset price growth and volatility vanishes if $p'(\eta) = 0$.

the price. However, the amplification mechanism depends on the under-insurance of experts. Non-contingency of debt is a crucial assumption that leads to under-insurance. With inside equity dependent on observable aggregate shocks such as price shocks, a state-variant capital requirement may provide partial insurance against aggregate risk and potentially alleviate fire sales and their effects.[57]

B.3 Margin based Asset Pricing and Risk Measure

In the previous subsection in this appendix, we considered how changes in the borrowing constraint can cause fire sales and asset market feedback. In this section, we consider the asset pricing implications of borrowing constraints. The question we aim to answer is: how capital requirements constraint affect the equity premium earned by experts? Intuitively, we can think of the equity premium as the monetary compensation to encourage experts to invest in the risky asset instead of the riskless. Theories of risk-based asset pricing postulate that the equity premium is completely explained by risk factors, notably market "beta", book-to-market ratio and market capitalization.[58]

Prevalent assumptions of these theories include the absence of arbitrage, borrowing or short-selling constraints. In the presence of borrowing constraints, equity premium reflects conventional risk premium and margin premium. As a result, two assets with completely identical cash flows may yield different equity premium, contrary to the law of one price. In most cases, theories assume segmented markets where not all agents can trade in both markets or impose trading restrictions in the form of margin requirements. For instance, Garleanu and Pedersen (2011) call this margin premium in returns; the more difficult it is to fund (i.e., the higher the margin or haircut), the higher is the required yield. Similarly, Fostel and Geanakoplos (2008) call this excess premium the collateral value of the asset; the easier it is to use the asset as collateral, the higher the price and lower the required premium. In contrast, the excess premium here is determined by the *joint* contribution of prices, internal funding costs and salience of market losses. For this reason, we further refer to it as the *salience loss* premium.[59] Therefore, regulatory capital requirement affects the experts' equity premium decomposition, directly through leverage and salience of market losses and implicitly through the tightness of the borrowing constraint.

[57]Fire-sale externalities often provide a rationale for financial regulation and capital requirements. These externalities arise when borrowing constraints that depend on endogenously determined asset prices are binding. Intuitively, agents do not internalize that their decisions affect collateral values through price changes, changing the effective borrowing capacity of other credit constrained agents. If the insurance market is missing, then fire-sale externalities generate over-investment Geanakoplos and Polemarchakis (1986). The government can substitute for missing insurance markets by influencing prices either by ex-ante capital requirements, and therefore reducing ex-ante investment, or by ex-post liquidity provision and asset purchases to substitute for missing demand. We abstract from the normative analysis and unconventional monetary policy, however, it is a possible promising extension. As recently showed by Davila (2011), the existence of amplification is not what necessarily causes the externalities, and the implications of borrowing constraints for amplification and welfare must be studied separately.

[58]For more details, see Capital Asset Pricing Model and Fama and French (1993) three-factor model.

[59]In the equation (7), salience loss premium equals $\xi(\eta)p(\eta)M(\eta)$.

C General Method of Moments estimation

In this section, we briefly provide results of general methods of moment estimation of asset pricing equation of banks that were recapitalized during the 2008 market downturn. In particular,we estimate coefficient b in the asset pricing equation from the main text(52).

$$E\left[\left(\frac{A}{p_t} + \mu_t^p + \sigma\sigma_t^p - r_t - \frac{1}{M_t}(\sigma + \sigma_t^p)^2 - \xi p_t M_t\right) X_t\right] = 0 \tag{52}$$

where

$$M_t = 1 - \mathrm{e}^{(\mu^p + \sigma\sigma^p) + b(\sigma + \sigma^p)} \tag{53}$$

are subjective value of market losses using Wang's probability weighting function. The time period we consider is September-December 2008. We use five Fama-French risk factors as instruments X_t for restricted GMM, and Y denotes if an instrument is used in estimation. p-value of Hansen J statistics is provided as a guidance for model selection.

Variable	Empirical counterpart	Source
Core Variables		
$\frac{A}{p_t} + \mu^p$	Holding period returns with dividends	CRSP
σ	VIX (The CBOE Volatility Index)	OptionMetrics
r_t	Treasury bill rate	Fama-French Data Library
σ_t^p	Historical volatility	WRDS
p_t	Equity price (Bid/Ask)	CRSP
Instruments		
SMB	Small Minus Big is the average return on the nine small stock portfolios minus the average return on the nine big stock portfolios	Fama-French Data Library
HML	High Minus Low is the average return on the two value portfolios minus the average return on the two growth portfolios	Fama-French Data Library
RMW	Robust Minus Weak is the average return on the two robust operating profitability portfolios minus the average return on the two weak operating profitability portfolios	Fama-French Data Library
CMA	Conservative Minus Aggressive is the average return on the two conservative investment portfolios minus the average return on the two aggressive investment portfolios	Fama-French Data Library
$Rm - Rf$	the excess return on the market, value-weight return of all CRSP firms incorporated in the US minus the one-month Treasury bill rate (from Ibbotson Associates)	Fama-French Data Library

Table 6: Restricted GMM estimation of Wang's risk spectrum during the crisis

	Asset Pricing Equation				
	(1)	(2)	(3)	(4)	(5)
b	-1.132^{***}	-1.199^{***}	-1.359^{***}	-1.188^{***}	-1.216^{***}
	(0.200)	(0.213)	(0.251)	(0.220)	(0.269)
xi	-0.480^{*}	-0.144^{***}	-0.159^{***}	-0.176^{***}	-0.103
	(0.246)	(0.043)	(0.037)	(0.057)	(0.070)
$R_m - R_f$		Y	Y		
SMB	Y				
HML	Y	Y		Y	Y
RMW			Y	Y	
CMA					Y
Observations	3,013	3,013	3,013	3,013	3,013

Notes: ***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level.