Macroeconomics with Financial Sector Risk Constraints

Branka Matyska*

October 11, 2019

Abstract

This paper presents a two-period simple macro model with the financial sector optimizing under the risk-based capital requirements. The goal of the optimal prudential policy is to maximize welfare by encouraging or discouraging risk-taking but to accomplish this objective through market risk measures and deposit insurance design. If fixed deposit insurance is unavailable, the optimal Value at Risk policy includes countercyclical loss probabilities. With deposit insurance, optimal capital requirements are higher in comparison to VaR capital regulation. Moreover, the optimal policy is procyclical or countercyclical depending on creditors' risk aversion. We also find that the Expected Shortfall embedded in the Basel III is optimal if creditors are risk-neutral and insurance regime with a fixed fee and variable compensation is provided. Our main message is that providing deposit insurance is suboptimal in terms of welfare levels in the absence of significant social costs of bank failure. Comparing different insurance regimes, we find that it is optimal for regulators not to neglect tail market outcomes when creditors are protected by deposit insurance.

*CERGE-EI (Center for Economic Research and Graduate Education - Economics Institute). email : Branka.Matyska@cerge-ei.cz

1 Introduction

The importance of welfare-improving risk management policies has never been greater. At the core, the recent financial crisis has emphasized that the widespread failures and losses of financial institutions and risk management policies can have severe macroeconomic consequences. According to Kashyap et al. (2008) and Mizen (2008), losses on mortgage-backed securities of investment banks propagated through the interbank lending market, causing a credit crunch and spillovers to the wider economy. Recent estimates suggest a remarkable 25% of the output loss to the recent 2008 financial crisis (Laeven and Valencia (2010)). Moreover, the great recession had a large adverse impact on the labor market as reported by Elsby et al. (2010). As suggested by Duffie (2019), essentially all relevant authorities agree that the existing regulation and supervision allowed large intermediaries to have insufficient capital and liquidity in comparison to the risk they held on their balance sheet. In addition, due to highly leveraged financial institutions, especially the too-big-to-fail investment banks, shocks can be severely amplified through the fragile financial system and have macroeconomic effects.

In this paper, we endeavor to bridge the gap between economic micro-founded theory and the actual regulations embedded in a form of the statistical measure of risk to study the optimal prudential policy. The questions we address are: What are the implications of the financial regulation adopted in the Basel framework? How should regulators measure market risk and design capital requirements from a welfare perspective? How should the optimal policy be implemented if deposit insurance is available or nonexistent? Therefore, the goal of the paper is twofold. From a positive perspective, the goal is to analyze the macroeconomic implications of capital requirements implemented as Value at Risk(VaR) risk measure. From a normative perspective, the goal is to develop the optimal market risk measure from the welfare point of view, which maximizes the representative households expected utility.

To do so, we consider the two-period version of Gertler and Kiyotaki (2010) model of government credit policies in the recent financial crisis. However, the goals of the paper and the nature of the financial frictions differ. More specifically, in Gertler and Kiyotaki (2010) bankers exhibit a moral hazard behaver and can divert a fraction of funds, which potentially constrains the ability of intermediaries to obtain funds from depositors. In Gertler and Kiyotaki (2010), the goal is to evaluate the efficiency of ex-post credit policies in the crisis resolution. In our model, we focus on ex-ante optimal prudential policies. Moreover, the capital requirement constraint in the form of a risk measure is imposed on intermediaries. Regulators perform two roles in our economy. First, they impose the minimum capital requirement constraint on banks, which is always binding. When solving for an optimal policy we consider two regimes. In the first regime, fixed or variable deposit insurance is available, whereas in the second regime deposits are uninsured. In the deposit insurance regime, regulators impose fixed insurance fee in order to repay depositors in case of bank insolvency.

Turning to the literature, the paper is connected to recent macroeconomic models which embed financial intermediaries and focus on the role of the banks in developing the crisis and the feedback from financial frictions onto economy(Gertler and Kiyotaki (2010), He and Krishnamurthy (2014), Brunnermeier and Sannikov (2014)). The vast majority in this literature concentrates on balance sheet channels via different financial frictions such as moral hazard or bank collateral constraints. Conversely, in this paper, we focus on capital requirements in the form of a risk measure. Our paper is complementary to papers focused on the design of capital regulation and its incorporation into macroeconomic models. Capital requirements have gained popularity in the crisis aftermath, as the severity of the crisis has opened the debate on the efficiency of prudential instruments(Freixas et al. (2015)). Most of the literature on prudential policy and policymakers have focused on countercyclical capital requirements as an efficient instrument to limit procyclicality of the financial sector. Currently, Basel III uses the credit-to-GDP gap as an early warning indicator of financial vulnerability and for setting countercyclical capital buffers(Committee et al. (2010)). From a positive perspective, the benefits of countercyclical requirements as prudential policies are analyzed,more recently, in Drehmann et al. (2010) and Repullo and Suarez (2012), and Repullo and Suarez (2012). For instance, Drehmann and Gambacorta (2012) find that countercyclical buffers can limit credit procyclicality. In contrast, Repullo and Saurina Salas (2011) argue against the proposal of using the credit-to-GDP gap, and recommend GDP growth as a guide for setting capital requirements instead. Whether optimal capital requirements are procyclical or countercyclical is determined endogenously in our model.

We depart in several ways from recent contributions that study prudential policies from a positive and normative perspective. As an important example of optimal prudential policies, Collard et al. (2017) derive jointly optimal monetary and prudential policies, setting the interest rate and capital requirements. As in our model, the optimality of countercyclical or procyclical capital buffers arises endogenously. Distinct to their model, the central bank has the role of a deposit insurance provider instead of enforcing interest rate policy rule. Repullo and Suarez (2012) study the implications of capital requirements embedded in Basel I, II and III on credit supply and welfare. Their analysis of the impact of Basel II capital requirements is analogous to our analysis of the implications of VaR capital regulation. Distinct than our model, Repullo and

Suarez (2012) impose full deposit insurance and allow for voluntary capital buffers. The second assumption means that banks may choose to hold capital above the minimum regulatory requirements in anticipation of future funding difficulties and raising equity capital in the future. In this way,Repullo and Suarez (2012) also capture the precautionary motive of banks independent of precautionary motives of regulators. As in Gertler and Kiyotaki (2010), we retain the assumption that bank equity is fixed in the short-run, reflecting the observation that bank equity is extremely costly to raise. Their second assumption implies that depositors are fully protected by deposit insurance and receive the fixed rate of return with probability one. This implies, distinct from our model, that depositors' utility is irrelevant for aggregate welfare. Moreover,Repullo and Suarez (2012) abstract from demand-side feedback effects. Our model is simple enough to allow us to incorporate the feedback from depositors to banks at the expense of omitting the richer firm-bank relationship dynamics as in Repullo and Suarez (2012). [1]

The qualitative implications of capital requirements are relevant for welfare purpose; however, in their design, we embrace a different approach. The quantification of the downside risk of a financial position is a key issue, both for financial institutions and regulators. The fundamental purpose of capital requirement policies is to quantify the downside market risk arising from banks holding the risky asset on their balance sheets. Among the papers which are close to our paper in terms of the statistical tail-behavior of financial institutions' asset returns in equilibrium are Adrian and Brunnermeier (2011), where CoVaR is proposed as an optimal systemic risk measure, and Acharya (2009) where the systemic expected shortfall is recommended instead. However, our emphasis is on the optimal market risk measure instead of systemic risk measures. The two measures are interrelated; as with an inadequate market risk measure, regulators can underestimate market risk, which in turn can lead to misspecification of systemic risk.

Most capital regulation is designed to ensure the solvency and soundness of individual financial institutions. When seen in isolation, capital requirements are designed to limit the probability of large market losses up to a small but acceptable threshold. Prior to the great recession, the Basel Committee used VaR to require financial institutions to meet capital requirements to cover the market risk that they incur as a result of the daily portfolio adjustments. From the early 90s, VaR has become the prevailing measure of market risk which answers the question: what is the maximum market loss

[1] The agency problem between banks and firms in Repullo and Suarez (2012) is a relevant factor for bank capital structure and regulation. In general, banks' equity financing is more costly than financing by deposits, so regulators may face a challenge of forcing banks to hold more equity than desired by banks (Kashyap et al. (2008)).

within a specified confidence interval? VaR was developed in order to provide senior management with a single number that can in a simple manner incorporate information about portfolio risk. Value at Risk may penalize diversification and it neglects tail losses that may hide behind the threshold. The post-crisis measure of risk, Expected Shortfall or conditional VaR, measures average losses above *VaR* threshold. However, market risk measures in Basel II and III are designed to limit the financial institution's risk seen in isolation, without taking welfare effects into account. Unlike our policy-oriented and welfare objective, most of the risk management literature is concerned with various proposals of new market risk measures from a portfolio optimization perspective. For example, Expected Shortfall, distortion risk measures and spectral risk measures are suggested to obtain the risk-return tradeoff instead of Markovitz's optimal portfolio approach (Wang (2000),Acerbi and Tasche (2002),Acerbi (2002)).

Our model highlights the potential procyclical effect on the supply of bank credit, as well as a microprudential role of capital requirements such as deposit insurance payouts due to bank failures, and its macroprudential role such as welfare. We find that under VaR capital regulation in the Basel II, financial intermediaries accumulate more risk when fundamentals are strong, that is when the expected return on assets on their balance sheet is high and volatility is low. Relevant to our result, the empirical evidence confirming VaR as a significant driver of procyclical leverage has been provided by Adrian and Shin (2010). Whether leverage is procyclical or countercyclical may have different implications for the optimal policy.

When deposit insurance is unavailable, the optimal policy implies less procyclical capital requirements by adjusting the confidence level of VaR. In practice, regulators implement this policy by allowing for a lower probability of bank default in the times of high economic growth and low uncertainty. In other words, a lower probability of market loss is allowed in market booms than in downturns. At impact, optimal policy resembles smoothing of the credit cycle. By comparison, under deposit insurance regime with fixed fee and compensation provided by the prudential or monetary authority, optimality of procyclical capital requirements does not necessarily carry over. Provided that creditors are sufficiently risk averse, the optimal policy is countercyclical. If creditors' risk aversion is smaller than one, the optimal policy is procyclical. At the same time, capital regulation mandates imposing higher capital ratios both in good and bad times. Our analysis also implies that with fixed deposit insurance, optimal market risk measure is inversely proportional to average expected *utility* of tail market outcomes beyond VaR threshold, which we name the inverse-Expected Utility Shortfall(i-EUS). Another important finding is that adopting the Expected Shortfall implemented in the Basel III is justified when creditors are risk neutral and their time preference rate equals the expected market return. Importantly, deposit insurance is a mixture of fixed fee and variable compensation. In sum, ES is optimal when creditors pay a fixed insurance fee to insure deposits, but in case the bank defaults compensation is paid per unit of invested deposits. We also find that bank credit supply is more procyclical under the risk-based requirements of Basel III than under the of Basel II. Our main message of the paper is that providing deposit insurance is suboptimal in terms of welfare levels in the absence of significant social costs of bank failure. Comparing different insurance regimes, we find that it is optimal for regulators not to neglect tail market outcomes when creditors are protected by deposit insurance. This finding suggests that once the government provides insurance, either the Expected Shortfall or the inverse-Expected Utility Shortfall dominates VaR as a market risk measure. The dominance is because with i-EUS as a market risk measure creditors are indifferent whether or not to buy insurance, while with ES creditors are indifferent whether the bank fails or not. We find that adopting ES is beneficial as long as welfare spillovers from market instability are the major concern for regulators. If, however, regulators aim to discourage banks to take high-risk investments while absorbing market gains, i-EUS may be an adequate choice instead. Overall, i-EUS may be a valuable policy in the pre-crisis years, while regulators could switch to ES once the financial crisis materializes.

The remainder of the paper is organized as follows. Section 2 presents the benchmark macro model without financial regulation. Section 3 gives the quick overview of risk measures incorporated in preceding and current financial regulations, specifically value at risk and expected shortfall. Section 4 presents a macro model with bank risk measurement and portfolio optimization under risk constraints. Further, it derives optimal risk measure from a welfare perspective with insured and uninsured deposits and provides a possible rationale for the expected shortfall embedded in Basel III. Section 5 concludes. Proofs and mathematical definitions are provided in appendices.

2 A macro model without financial regulation

2.1 Households

In this section, we present an equilibrium model, which we subsequently modify to include ex-ante capital requirement constraint. Importantly, the benchmark model is deterministic, while the model with regulation is stochastic due to market risk. The implicit reason for regulating banks is that their undercapitalization can adversely affect aggregate welfare. The benchmark model is a simplified two-period version of Gertler and Kiyotaki (2010). The representative household consists of two types of members:

bankers and workers. There are two periods in the model. In the first period, workers are endowed with y units of consumption good which they allocate between deposits they put in the bank and consumption. The first-period budget constraint is equal to

$$c + d \leq y, \tag{1}$$

where d is the amount of deposits households supply to the bank, and c household consumption in the first period. In the second period, household only consumes its income, i.e. wealth which is equal to the sum of gross return on deposits they invested in the first period, and the profits bankers bring to the household. The second-period budget constraint is

$$C \leq R^d d + \pi, \tag{2}$$

where R^d and π are the gross return on deposits and the profit brought by bankers, respectively. The household chooses d, taking R^d and π as given, in order to maximize its lifetime utility

$$u(c) + \beta u(C) \tag{3}$$

subject to (1) and (2). The first order condition with respect to d implies

$$u'(c) = \beta R^d u'(C),$$

which gives us the classical Euler intertemporal substitution equation. After the second period, the household dies and has no utility of consuming. Euler equation reads that higher interest rate R^d implies a higher willingness of household to substitute towards tomorrow's consumption. Since the utility function is increasing in consumption, the second-period budget constraint is binding. Substituting for deposits d from the second into a first-period budget constraint, we obtain the intertemporal budget constraint

$$c + \frac{C}{R^d} \leq y + \frac{\pi}{R^d}. \tag{4}$$

The left-hand side of the intertemporal budget constraint corresponds to the present discounted value of household consumption, while the right-hand side denotes the present discounted value of a household's income. The condition says that the first must be less or equal to the latter, as households cannot die in debt. Let the household preferences be given by CRRA utility with relative risk aversion factor γ

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

Then, the first order condition gives us

$$c^{-\gamma} = \beta R^d C^{-\gamma}.$$

Substituting for C from the first-order condition into the intertemporal budget constraint we get first-period consumption

$$c = \frac{y + \frac{\pi}{R^d}}{\frac{(\beta R^d)^{\frac{1}{\gamma}}}{R^d} + 1}.$$

We can see that the household chooses first-period consumption as a fraction of its wealth, which is a decreasing function of the interest rate paid on deposits for $\gamma \leq 1$. Hereafter, we impose $\gamma \leq 1$. Substituting for c in the first-period budget constraint, we obtain a household's supply of deposits

$$d = y - c = y - \frac{y + \frac{\pi}{R^d}}{\frac{(\beta R^d)^{\frac{1}{\gamma}}}{R^d} + 1}.$$

2.2 Firms

There is a continuum of competitive firms in the economy. Unlike workers and bankers, firms have no endowment and need to issue security s in the first period to finance the purchase of goods to produce capital, with one-to-one technology

$$s = k. \tag{5}$$

In the second period, firms produce goods from capital using a linear production function

$$f(k) = R^k k, \tag{6}$$

where R^k is a gross return on capital which is certain and equal to μ. Notice that we implicitly assume that the firm production function has constant returns to scale. Since firms are competitive, they earn no profit, so the return on a security is equal to the return on capital.

2.3 Banks

Banks play two roles in the benchmark model: they provide credits to end-borrowers, i.e. firms and provide liquidity in the form of deposits to households. In the extended model with regulation, banks also provide partial insurance against market risk. In the first period, bankers are endowed with N units of net worth. Hereafter, we will use terms equity, bank capital, and net worth interchangeably. In a dynamic model, equity would represent retained earnings from the previous period and could be adjusted continually. However, in our model bankers cannot issue equity to finance asset purchases from firms but can only take deposits from households and combine them with net worth to purchase securities.

$$s = N + d^b. \tag{7}$$

At this point, we distinguish between household's supply of deposits d and bank's demand for deposits d^b. In equilibrium, due to the market clearing condition, the two will be the same. As seen from (7), in the first period, the bank's portfolio consists of riskless security on the asset side and equity and deposits on the liability side. In the second period, the bank pays off a deposit to the household and earns a profit

$$\pi = R^k s - R^d d^b. \tag{8}$$

The bankers are risk neutral and aim to maximize profit by choosing s and taking R^d as given.

2.4 Benchmark equilibrium

We now define the benchmark equilibrium, where regulators do not impose capital requirement constraint on banks.

Benchmark equilibrium : The equilibrium consists of values of $c, C, R^d, d, d^b, \text{and} \pi$ such that

1. the household's maximization problem is solved,
2. the bank's maximization problem is solved,
3. market for deposits clears, $d = d^b$,
4. market for securities clears, i.e. (5) holds.

Here, we consider only interior equilibrium, in which $c, C, d, d^b > 0$. For interior equilibrium to exist we must have that

$$R^d = R^k \equiv \mu. \tag{9}$$

This can be easily seen from the bank's problem. If $R^d < \mu$, then the bank would want to borrow an infinite amount, $d^b = \infty$, which exceeds the household endowment in the first period. Similarly, if $R^d > \mu$, the bank would set deposits equal to zero, which violates the interior equilibrium requirement. To solve for equilibrium value of deposits, we impose $d = d^b$ and substitute profits in (8) into the household's supply of deposit

$$d = y - c = y - \frac{y + \frac{\pi}{R^d}}{\frac{(\beta R^d)^{\frac{1}{\gamma}}}{R^d} + 1} = y - \frac{y + \frac{\mu s - R^d d}{R^d}}{\frac{(\beta R^d)^{\frac{1}{\gamma}}}{R^d} + 1}$$

$$= y - \frac{y + \frac{\mu(N+d) - R^d d}{R^d}}{\frac{(\beta R^d)^{\frac{1}{\gamma}}}{R^d} + 1} = \frac{y(\beta R^d)^{\frac{1}{\gamma}} - \mu N - d(\mu - R^d)}{(\beta R^d)^{\frac{1}{\gamma}} + R^d}.$$

Solving for d we get

$$d = \left(1 + \frac{\mu - R^d}{(\beta R^d)^{\frac{1}{\gamma}} + R^d}\right)^{-1} \frac{y(\beta R^d)^{\frac{1}{\gamma}} - \mu N}{(\beta R^d)^{\frac{1}{\gamma}} + R^d} = \frac{y(\beta R^d)^{\frac{1}{\gamma}} - \mu N}{(\beta R^d)^{\frac{1}{\gamma}} + \mu}. \tag{10}$$

Substituting (10) into the first-period budget constraint (1) and the second-period constraint in (2), we obtain household's consumption in the first period

$$c = y - d = y - \frac{y(\beta R^d)^{\frac{1}{\gamma}} - \mu N}{(\beta R^d)^{\frac{1}{\gamma}} + \mu} = \frac{\mu(y+N)}{(\beta R^d)^{\frac{1}{\gamma}} + \mu}, \tag{11}$$

and consumption in the second period

$$C = R^d d + \pi = R^d d + R^k s - R^d d = R^k(N+d) = \mu \frac{(\beta R^d)^{\frac{1}{\gamma}}(y+N)}{(\beta R^d)^{\frac{1}{\gamma}} + \mu}. \tag{12}$$

The benchmark equilibrium is defined by equations (9)-(12).

3 VaR and Expected Shortfall as market risk measures

In this section, we elucidate different measures of market risk in the trading book of financial institutions. Recent accords of the global financial regulation, namely Basel II and III, have adopted different measures of market risk: conventional VaR and Expected Shortfall. Both measures are designed to quantify portfolio risk of an uncertain financial position[2]. These risk measures are used to decide required regulatory capital for a given portfolio, based on its downside risk potential. A popular risk measure for capital requirements in the banking industry VaR is based on a quantile concept. In essence, VaR is the maximum loss with a specified confidence interval. In other words, VaR measures f how much a certain portfolio can lose within a given time horizon, for a given confidence level. From shareholders' or management's perspective, the quantile VaR at the company level is a meaningful risk measure since the default event itself is of primary concern, and the size of a shortfall is only secondary. On the other hand, Expected Shortfall measures average losses exceeding VaR limit, that is the average expected size of a shortfall.

While VaR measures the worst market losses which can be expected with a small probability, VaR is heavily criticized for violating subadditivity and monotonicity. Subadditivity means that the risk of a portfolio can be larger than the sum of individual risks of portfolio positions measured by VaR. Subadditivity reflects the notion that individual risks typically diversify (or, at worst, do not increase) when we put risky positions together. As a response to this deficiency, the notion of *coherent* risk measure was introduced in Artzner et al. (1999) by proposing four axioms a coherent risk measure should satisfy[3]. Intuitively, translation invariance and subadditivity require that adding safe asset into portfolio decreases the risk of a portfolio and that merging does not create additional risk, respectively. Further, positive homogeneity and monotonicity require that scaling portfolio scales risk and that there is no risk if a portfolio incurs no losses. In this interpretation, VaR is not a coherent risk measure since it may discourage diversification and thus violates subadditivity[4]. This implies that the portfolio risk of a portfolio can be larger than the individual risk of portfolio positions measured by VaR. On the other hand, Expected Shortfall is a coherent risk measure for any confidence level[5].

There is scarcity in the literature that explores a connection between coherent risk measures and expected utility theory. Notably, Acerbi (2002) offers a way to map any

[2]Both VaR and ES are moment-based risk measures. The predominant moment based risk measure is the standard deviation due to Markowitz (1952).

[3]See Definition A.1 in Appendix A.

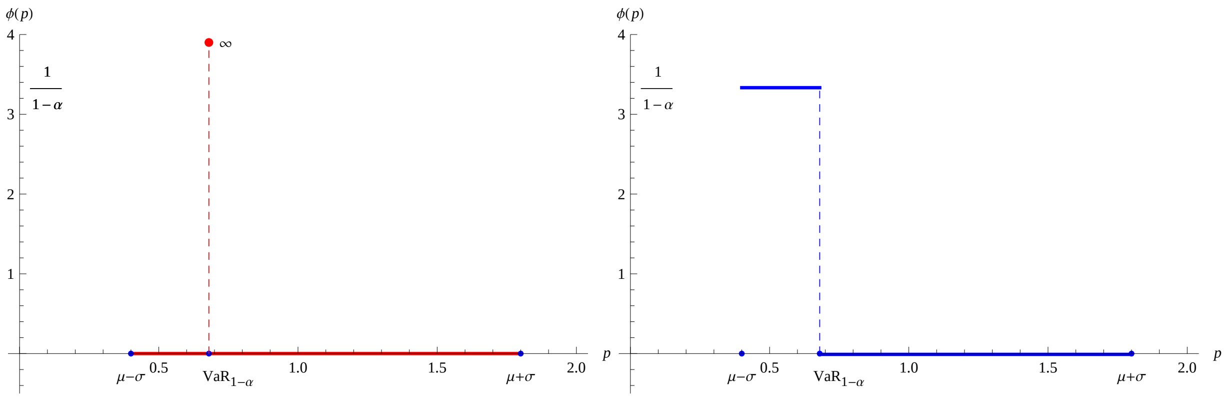
[4]See Acerbi and Tasche (2002) for the proof.

[5]See Acerbi and Tasche (2002) for the proof and definition of Expected Shortfall.

rational investor's subjective risk aversion function into a coherent risk measure and vice-versa. Intuitively, coherence of a risk measure means that larger weight is assigned to larger losses (worst market outcomes). Thus, coherency is attained if the weighting function, that is the subjective risk aversion function, is positive, decreasing and normalized to one on the unit interval. In this interpretation, both VaR and Expected Shortfall could be considered as a special case of a larger set of spectral risk measures. In the former case, the subjective risk aversion is infinity at a specified confidence level and zero elsewhere, and consequently, coherency is lost. The investor's risk attitude is incoherent, focusing on a threshold level of worst $(1-\alpha)\cdot 100\%$ losses and neglecting larger losses in the tail. In the latter case, equal weight is given to the threshold loss and all larger losses, and coherency is preserved. We refer to Figure 1a and Figure 1b for graphical representations.

We now define in more details two market risk measures, namely VaR and Ex-

Figure 1: Weighting function



(a) VaR (b) ES

pected Shortfall. Let X be a random variable the cumulative distribution function $F_X(x) = P[X \leq x]$, where X represents random gains or losses of a portfolio. Let us fix some confidence level $\alpha \in (0,1)$. We define VaR as the largest market loss that can occur with confidence level not smaller than α, that is

$$VaR_\alpha(X) = \min\{x| \quad F_X(x) \geq \alpha\} = q_\alpha(X),$$

where $q_\alpha(X)$ denotes the $\alpha-$quantile [6] of the distribution of market gains and losses. Note that $VaR(X)$ gives the size of losses below zero, which may occur with probability no greater than $1-\alpha$. On the other hand, Expected Shortfall with a confidence level α measures how large losses can be expected if the return of portfolio drops below its

[6]See Definition A.2 in the Appendix A.

$\alpha-$quantile. The following representation of $ES_\alpha(X)$ is given by Acerbi and Tasche (2002)

$$ES_\alpha(X) = \frac{1}{\alpha}\int_0^\alpha VaR_\tau(X)d\tau = E[X|X \leq VaR_\alpha(X)].$$

In the following section, we incorporate the above definitions of two market risk measures in the equilibrium model with market risk and financial regulation.

4 Macro model with financial regulation : Value at Risk

After computing the benchmark equilibrium, we now introduce financial regulation in the form of a risk measure that changes the bank problem. To do so, we modify the bank's problem so that bankers can earn a profit and use Value at Risk constraint to manage market risk. For these reasons, the return on capital investment is now uncertain. We can think of the risky security as granting direct loans to private firm borrowers, but there is a default risk that the borrower will fail to honor his loan obligations. The risky security is traded in the first period in anticipation of its realized return in the second period. Let us make a simplifying assumption that return on capital, R^k, is uniformly distributed over the interval

$$[\mu - \sigma, \mu + \sigma],$$

where both μ and σ are known by the household and the bank. The mean and the variance of R^k are given by

$$E(R^k) = \mu,$$

$$Var(R^k) = \frac{\sigma^2}{3}.$$

Since the return on capital is uncertain, the bank's profit is uncertain when viewed from the first period. As a result, uncertainty in project outcome injects risk in the bank's balance sheet. In particular, we suppose that the banker now has two options, default and not default, where default occurs if the realized return is below $VaR_{1-\alpha}$ threshold. In both cases, in the first period the bank issues deposits d^b, combine these with a net worth N to purchase securities $s = N + d^b$. If the realized return is above the threshold, the bank pays $R^d d^b$ to the household and earns a profit of $sR^k - R^d d^b$. This option is the only one emphasized in the benchmark equilibrium. If the realized return is below the threshold, the bank does not pay $R^d d^b$ and stays with $R^k s$. Thus, the bank maximizes expected profit subject to the Value at Risk constraint. The question VaR answers is: what is the maximum loss with a specified confidence level? For a given confidence level α, for example, $\alpha = 0.9$, VaR is such that

$$Prob(R^k \leq VaR_{1-\alpha}) = 1 - \alpha \tag{13}$$

holds. This gives us the expression for the Value at Risk quantile

$$VaR_{1-\alpha} = F^{-1}(1-\alpha)$$

where $F^{-1}(\cdot)$ is the inverse of the cumulative distribution function of the return on capital R^k. With a uniformly distributed market return, $VaR_{1-\alpha}$ can be easily computed

$$\int_{\mu-\sigma}^{VaR_{1-\alpha}} \frac{1}{2\sigma} dR^k = 1 - \alpha$$

$$VaR_{1-\alpha} = \mu + (1-2\alpha)\sigma$$

and graphically represented in Figure 2.

Now let us consider how the Value at Risk constraint enters the bank maximization

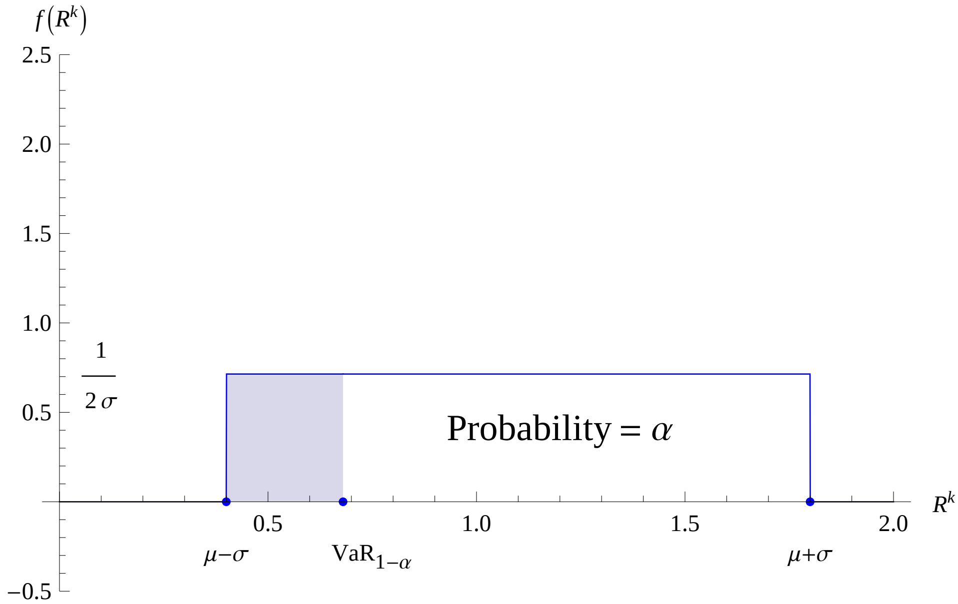


Figure 2: VaR of uniformly distributed market return

problem. Recall that in the benchmark equilibrium $\mu = R^d$. With regulation, since the bank is risk-neutral and the interest rate endogenous, if $\mu > R^d$, the bank would wish to hold a larger and larger position in the risky security. Value at Risk constraint will put the bound on the amount of risky security the bank can purchase. The bank wants to maximize the expected profit, subject to the constraint that the probability of being insolvent in the second period is kept at an acceptable level. Insolvency means that book equity in the second period is less or equal to zero. Since there are only two periods, second-period equity is equal to the difference between the market value of assets and the market value of liabilities. Let α be the fixed probability that the bank is solvent in the second period. Since the bank becomes insolvent if equity falls below zero, we have

$$Prob(R^k s - R^d(s-N) \leq 0) \leq 1-\alpha, \quad \text{or equivalently}$$

$$Prob(R^k \leq R^d(1-\frac{N}{s})) \leq 1-\alpha. \tag{14}$$

Therefore, by choosing the size of s, banks are assured that the probability of default is at most $1-\alpha$. Comparing the right-hand side terms in (13) and (14) inside the probability brackets we have that the bank becomes insolvent if

$$1-\frac{N}{s} \leq \frac{F^{-1}(1-\alpha)}{R^d}. \tag{15}$$

From a financial stability perspective, regulators aim to control risk-taking of banks by focusing on market loss absorbency of the balance sheet, which ensures partial solvency of banks balance sheets and the attendant systemic risks. In particular, both the probability of insolvency and the probability of market loss being higher than the VaR threshold are equal to α. In a dynamic model, fixing the probability of loss does not imply that insolvency probability is fixed. At the present moment, systemic risk is beyond the scope of this paper, even though systemic risk provides the prevailing rationale for regulating banks in the first place. By comparison, setting $\alpha = 1$ stipulates that the debt issued by the bank is risk-free and in that case, all implications would emanate from actions in anticipation of defaults, rather than from defaults themselves. Rearranging (15), we obtain the Value at Risk constraint

$$s - F^{-1}(1-\alpha)s/R^d \leq N. \tag{16}$$

The VaR constraint says that the worst possible loss relative to today's market value of securities must be met by today's equity. In other words, capital requirement constraint forces banks to keep a capital-to-loans ratio of at least $1-F^{-1}(1-\alpha)s/R^d$. The bank maximizes expected profit subject to the constraint

$$\begin{aligned} \max_s \quad & \mu s - \alpha R^d(s-N) \\ \text{s.t.} \quad & s - F^{-1}(1-\alpha)s/R^d \leq N. \end{aligned} \tag{17}$$

The Lagrangian representation of the bank's problem is

$$\max_{s} \quad \mu s - \alpha R^d(s-N) + \lambda(F^{-1}(1-\alpha)s/R^d + N - s).$$

The first order conditions associated with the banker problem in equilibrium are:

$$\begin{aligned}
[s]: &\quad \mu - \alpha R^d + \lambda(F^{-1}(1-\alpha))/R^d - \lambda = 0 \\
[\lambda]: &\quad \lambda((F^{-1}(1-\alpha))s/R^d + N - s) = 0,
\end{aligned}$$

where λ is the Lagrange multiplier associated with the constraint. If the constraint is not binding, we have that $\lambda = 0$. Substituting $\lambda = 0$ into $[s]$: equation, we obtain $R^d = \frac{\mu}{\alpha}$. But this would mean that the bank would earn no profit on deposits, and would be as well off as purchasing only amount N of securities. Since we are interested in cases when deposit demand is positive, the constraint is binding when $\lambda > 0$. As a result, profit maximization leads banks to choose the largest value of risky security holdings allowed by the Value at Risk constraint

$$s = \frac{N}{1 - \dfrac{F^{-1}(1-\alpha)}{R^d}}. \tag{18}$$

The demand for deposits is given by

$$d^b = s - N = N\frac{F^{-1}(1-\alpha)}{R^d - F^{-1}(1-\alpha)}. \tag{19}$$

4.1 Household

With imposed the VaR constraint on bankers, households know that banks operate on the probability of default and that it obtains the return on deposits only in the non-default state. Therefore, households maximization problem is

$$\begin{aligned}
&\max_{c, C^{ND}} \quad u(c) + \beta E(u(C^{ND})|\text{not default}) \\
&\text{s.t.} \quad c + d = y \\
&\text{s.t.} \quad C^{ND} = R^d d + \pi.
\end{aligned} \tag{20}$$

Since in the case of default household's consumption is equal to profit which is transferred exogenously to households from bankers, π does not alter the maximization problem. We are interested in solving for optimal household's deposit supply. Therefore, we rewrite optimization problem in terms of deposits

$$\max_{d} \quad \frac{(y-d)^{1-\gamma}}{1-\gamma} + \frac{\beta}{\alpha} \int_{\mu+(1-2\alpha)\sigma}^{\mu+\sigma} \frac{(R^d d + \pi)^{1-\gamma}}{1-\gamma} \frac{1}{2\sigma} dR^k.$$

Differentiating with respect to d we get the first-order condition

$$-(y-d)^{-\gamma} + \frac{\beta}{2\sigma\alpha} R^d \int_{\mu+(1-2\alpha)\sigma}^{\mu+\sigma} (R^d d + \pi)^{-\gamma} dR^k = 0.$$

Since in equilibrium profit transfered to the household is $R^d d + \pi = R^k(N+d)$

$$(y-d)^{-\gamma} = \frac{\beta}{2\sigma\alpha} R^d (N+d)^{-\gamma} \left(\frac{(\mu+\sigma)^{1-\gamma}}{1-\gamma} - \frac{(\mu+(1-2\alpha)\sigma)^{1-\gamma}}{1-\gamma} \right)$$

Denoting by $B = \dfrac{(\mu+\sigma)^{1-\gamma}}{1-\gamma} - \dfrac{(\mu+(1-2\alpha)\sigma)^{1-\gamma}}{1-\gamma}$, we get household's deposit supply

$$d = \frac{\left(\frac{BR^d\beta}{2\alpha\sigma}\right)^{\frac{1}{\gamma}} y - N}{1 + \left(\frac{BR^d\beta}{2\alpha\sigma}\right)^{\frac{1}{\gamma}}}, \tag{21}$$

first-period consumption

$$c = y - d = \frac{y+N}{1 + \left(\frac{BR^d\beta}{2\alpha\sigma}\right)^{\frac{1}{\gamma}}}, \tag{22}$$

and the total purchase of securities

$$s = N + d = \frac{(y+N)\left(\frac{BR^d\beta}{2\alpha\sigma}\right)^{\frac{1}{\gamma}}}{1 + \left(\frac{BR^d\beta}{2\alpha\sigma}\right)^{\frac{1}{\gamma}}}. \tag{23}$$

Equilibrium with Value at Risk : The equilibrium consists of values of c, C, R^d, d, d^b, andπ such that

1. the household's maximization problem is solved,

Table 1: Baseline parameter values

μ	σ	α	β	y	N
1.0629	0.5094	0.9417	0.95	1	0.2435

2. the bank's maximization problem is solved,

3. market for deposits clears, $d = d^b$,

4. market for securities clears, i.e. (7) holds.

In the following, we discuss the properties and implications of equilibrium with the VaR constraint. Table 1 describes our baseline parameters of the model. The discount rate β is set to 0.95, and we normalize the size of the non-banking sector y to 1. For the parameters pertaining to the banking sector, we follow Repullo and Suarez (2012). According to Repullo and Suarez (2012), an average *Total interest income* of all US commercial banks was 5.74% of *Earning assets*, *Total interest expense* was 2.32% of *Total liabilities*, and *Service charges on deposit accounts* were 0.55% in the pre-crisis years 2004-2007. This implies the intermediation margin of 3.97% on deposit-funded activities. Therefore, we set μ to 6.29%, while α and σ and σ are set such that the equilibrium interest rate is 2.32%, an average intermediation margin is 3.97% and loss given default parameter(LGD) is 0.45. The loss given default determines the loss of the loans of projects that fail, which is set according to the Basel II Internal Ratings-Based (IRB) foundation approach for unsecured corporate exposures. This calibration leads to a system of equations in N, α and σ, which has a unique solution with positive values of parameters. Solving for these parameters yields the steady-state default probability of 5.83% and the steady-state capital requirement of 40.1% of total assets.

In both equilibria with and without financial regulation, first-period consumption is a fraction of total endowment of the bank and the worker. Note that market risk and financial regulation affect this fraction through the choice of the confidence level α. In this way, both the interest rate and previously defined B are affected by the confidence level. We can think of B as per unit expected marginal utility of non-defaulted risky security. Higher is the confidence level, higher is utility. Expected marginal utility is further adjusted for the probability of not defaulting on debt and uncertainty of return distribution (bounded distribution support 2σ). We obtain the interest rate from the market clearing condition for deposits

$$N\frac{F^{-1}(1-\alpha)}{R^d - F^{-1}(1-\alpha)} = \frac{\left(\dfrac{BR^d\beta}{2\alpha\sigma}\right)^{\frac{1}{\gamma}} y - N}{1 + \left(\dfrac{BR^d\beta}{2\alpha\sigma}\right)^{\frac{1}{\gamma}}}. \tag{24}$$

For ease of exposition let us assume log-preferences. In this case, $\gamma = 1$, the equilibrium interest rate is equal to

$$R^d = \frac{2\alpha\sigma N + \beta B(\mu + (1-2\alpha)\sigma)(y+N)}{\beta B y}, \tag{25}$$

and $B = \log(\mu + \sigma) - \log(\mu + (1-2\alpha)\sigma)$. Recall that without market risk and regulation, the interest rate increases in a one-to-one fashion with respect to return on capital, $R^d = \mu$. Importantly, the choice of a risk measure affects the household's willingness to substitute between today's and tomorrow's consumption. With market risk and VaR imposed as a market risk measure, we see that B introduces nonlinear dependence of the interest rate on expected return and volatility. Moreover, it is increasing in former and decreasing in the latter. In other words, the interest rate is high when fundamentals are strong and uncertainty is low. This prediction of procyclical interest rate or borrowing costs is easily seen from

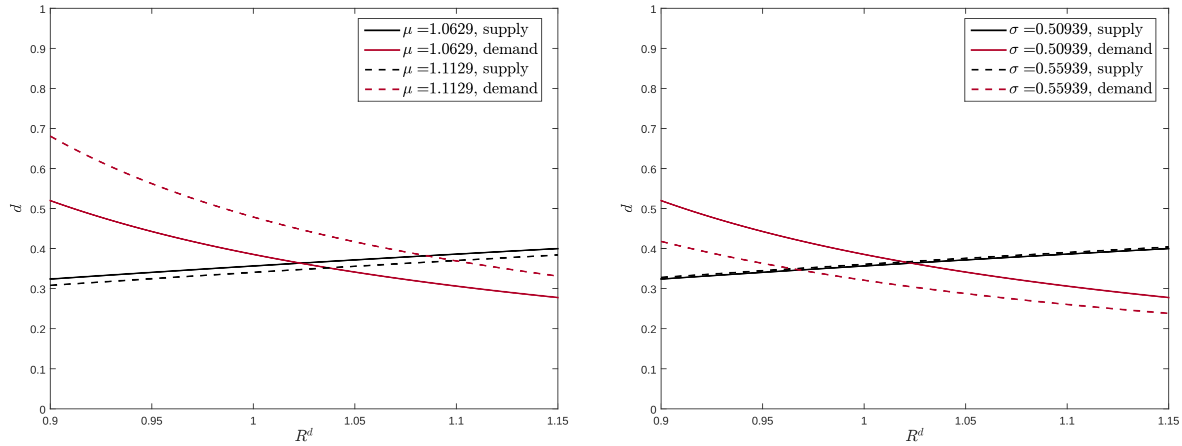
$$\frac{dR^d}{d\mu} = \frac{\dfrac{4\alpha^2\sigma^2 N}{(\mu+\sigma)(\mu+(1-2\alpha)\sigma)} + \beta(N+y)B^2}{\beta y B^2} > 0.$$

and displayed in Figure 3a and Figure 3b. The fact that the interest rate varies with expected return and volatility implies another distinction from the benchmark equilibrium, which is that we have a spread in equilibrium. Since the spread or the equity risk premium is defined as the difference between the expected return on capital and the interest rate, we have

$$\frac{d(\mu - R^d)}{d\mu} = 1 - \frac{dR^d}{d\mu} < 0,$$

since $\dfrac{dR^d}{d\mu} > 1$. We could rephrase this finding as saying that the equity risk premium under VaR regulation is countercyclical; the compensation banks demand to hold risky security increases when fundamentals are low. Another way we can think about the equity risk premium in the economy is through the Lagrange multiplier. The envelope

Figure 3: Equilibrium interest rate with VaR



(a) increase in expected return

(b) increase in volatility

theorem for constrained maximum gives us the expression and the interpretation of the Lagrange multiplier λ. From the envelope theorem, we know that

$$\lambda = \frac{dE(\pi)}{dN} = \frac{dE(\pi)}{ds}\frac{ds}{dN}$$

$$= \alpha\left(\mu - R^d\right)\frac{1}{1 - \dfrac{F^{-1}(1-\alpha)}{R^d}}.$$

$$= \alpha\left(\mu - R^d\right)\frac{1}{\dfrac{(2\alpha-1)\sigma - (\mu - R^d)}{R^d}}.$$

The Lagrange multiplier measures the increase in the objective function (expected profit) that is obtained through a marginal relaxation in the constraint (an increase in equity) and therefore can be interpreted as the shadow value of bank capital. From the above expression, we see that as the spread subsides, the Lagrange multiplier λ declines. This means that the return to a unit of bank capital diminishes as fundamentals appreciate in value.

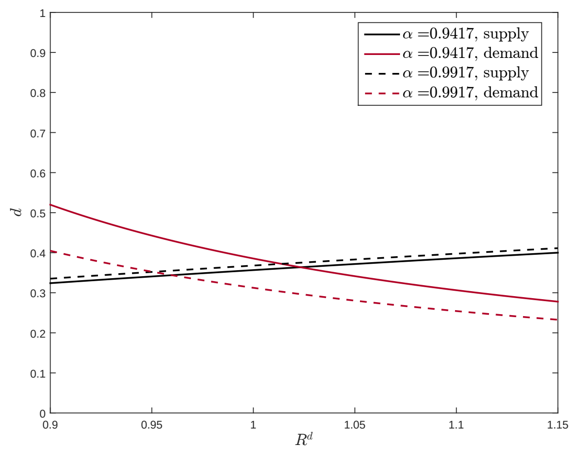
Another interesting property of the equilibrium interest rate is that it is increasing in the relative wealth of bankers, which can be easily seen from

$$\frac{dR^d}{d(\frac{N}{y})} = \frac{2\alpha\sigma + \beta B(\mu + (1-2\alpha)\sigma)}{\beta B} > 0$$

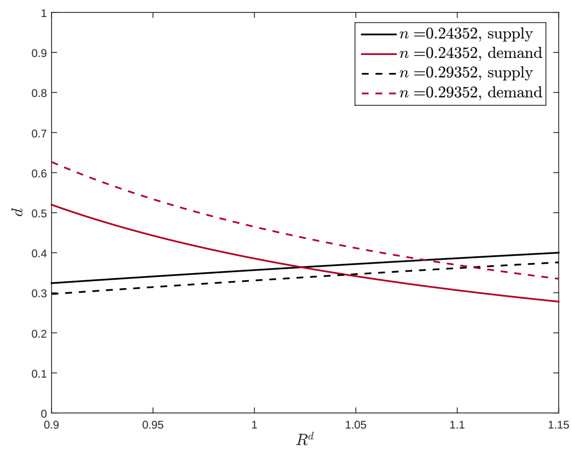
and Figure 4b. Holding everything else fixed, the more " skin in the game" banker has, the higher is the cost of borrowing. The prediction of the external financing costs being increasing in the strength of borrowers balance sheet is inconsistent with the financial accelerator in Bernanke et al. (1999). In addition, our prediction is in contrast with Gertler and Kiyotaki (2010) result where a financial accelerator emerges in equilibrium when bankers have incentives to divert funds. In their model with moral hazard, as banks balance sheet strengthens with improved economic conditions, the external finance problem declines. In our model, having larger equity conveys information that banks have incentives to engage in more risk-taking, which may induce households to reduce deposit supply and borrowing costs rise accordingly.

Our final result regarding the interest rate is about the effect of the insolvency probability. In the recent 2008 crisis aftermath, Gennaioli and Shleifer (2018) note that the two most important factors contributing to the crisis were flawed financial sector regulation and supervision and underestimated downside tail risks. In particular, they ascribe inaccurate beliefs about downside risk as to the main contributor and suggest that investors and regulators assigned unjustifiably low probabilities to disaster outcomes in the housing market. In our model, if regulators allowed for a lower probability of insolvency, this would reduce bankers' demand for risky security and increase households' supply. As a result of inelastic supply and more elastic demand adjustments, the equilibrium interest rate declines (Figure 4a). Overall, we find that changes in fundamentals, uncertainty, and insolvency have distinct effects on households' and bankers' deposit choice. We summarize these results in the following propositions and present proofs in Appendix A.

Figure 4: Equilibrium interest rate with VaR



(a) increase in confidence level



(b) increase in bank capital

Proposition 1. *The interest rate on deposits is increasing in μ and decreasing in σ.*

Proposition 2. *Leverage of the banking sector is procyclical.*

Proposition 3. *Leverage of the banking sector is decreasing in volatility.*

Since we focus on an interior equilibrium where $d^b > 0$, the bank is leveraged in the sense that total assets on the balance sheet are larger than the size of equity. Leverage is defined as the ratio of total assets to equity, that is

$$L = \frac{s}{N}. \tag{26}$$

Substituting for s we can compute leverage as

$$L = \frac{N/N}{1 - \dfrac{F^{-1}(1-\alpha)}{R^d}} = \frac{R^d}{R^d - F^{-1}(1-\alpha)}. \tag{27}$$

It is straightforward to see from (27) that if bank's funding costs are constant, leverage is increasing in the expected return and decreasing in volatility [7]. In this model, we get that result when also accounting for the endogenous borrowing costs. Having balance sheets marked to market through the VaR constraint, banks adjust leverage in response to changes in fundamentals. To elaborate more on that, from Appendix A we know that the sign of changes in bank leverage with respect to changes in the expected return on capital $\frac{dL}{d\mu}$ depends on the sign of the following expression

$$R^d - F^{-1}(1-\alpha)\frac{dR^d}{d\mu}.$$

After substituting for R^d and $\frac{dR^d}{d\mu}$ and rearranging, it is easy to show that the sign of $\frac{dL}{d\mu}$ is positive and proportional to

$$\frac{2\alpha\sigma N((\mu+\sigma)B - 2\alpha\sigma)}{\beta B^2(\mu+\sigma)y} > 0.$$

This implies that the leverage of the banking sector is procyclical under VaR regulation, i.e. leverage rises as the expected return on capital increases. In contrast, in Gertler and Kiyotaki (2010), leverage is countercyclical. The empirical importance of VaR as a driver of procyclicality of leverage has been emphasized by Adrian and Shin (2010). From Proposition 3 we know that the change in volatility influences bank's behaviour through changes in leverage. Although an increase in volatility has a negative effect

[7] Recalling that $F^{-1}(1-\alpha) = VaR_{1-\alpha} = \mu + (1-2\alpha)\sigma$.

on the interest rate, its effect on leverage depends on the sign of

$$(1-2\alpha)R^d - F^{-1}(1-\alpha)\frac{dR^d}{d\sigma} = \frac{-2\alpha\mu N((\mu+\sigma)B - 2\alpha\sigma)}{\beta B^2(\mu+\sigma)y} < 0,$$

which is negative. In summary, leverage of the banking sector is higher when fundamentals are strong and uncertainty is low and vice versa, lower leverage is associated with weaker fundamentals and higher uncertainty. This can be easily seen from Figure 3a and 3b, as the equilibrium level of deposits slightly rises and falls with positive changes in expected return and volatility respectively. In summary, we find that under VaR regulation banks accumulate risk in the form of higher leverage in periods of sustained growth and peaceful times.

4.2 Optimal policy with uninsured deposits

So far, the confidence level α was fixed and predetermined by regulators. In this section, we aim to answer the question of how should the insolvency or loss probability vary with changes in μ and σ. In other words, how should the optimal prudential policy from a welfare perspective adjust to changes in expected return and uncertainty? Importantly, we assume that regulators still impose VaR constraint on banks when implementing optimal policy. However, unlike in the previous section, the loss probability is allowed to adjust to different economic environments. We further assume that a social planner has no possibility to provide deposit insurance to either households or banks, and therefore deposits are uninsured. In order to find the optimal policy, we solve the social planner's problem whose aim is to maximize expected utility

$$\max_{\alpha} \quad u(c) + \beta E u(C^{ND}|\text{not default}) \tag{28}$$

by choosing d subject to two resource constraints

$$c = y - d,$$

$$C^{ND} = R^k(N+d).$$

With log-preferences the maximization problem simplifies to

$$\max_{d} \quad \log(y-d) + \beta \int_{\mu+(1-2\alpha)\sigma}^{\mu+\sigma} \log(R^k(N+d))\frac{1}{2\sigma}dR^k,$$

which gives

$$\max_{d} \quad \log(y-d) + \beta\alpha \log(N+d) + const.$$

Differentiating with respect to d and solving FOC we get

$$d^{SP} = \frac{\beta\alpha y - N}{\beta\alpha + 1}.$$

Notice that the social planner internalizes profit transferred to households from bankers, and therefore the optimal deposit choice does not depend on the interest rate. Furthermore, only the probability of loss is relevant for the social planner. The expected return and uncertainty of the risky security do not influence deposit choice. Recall from the previous section that we obtain the optimal amount of deposits d from competitive equilibrium after substituting for R^d. We obtain

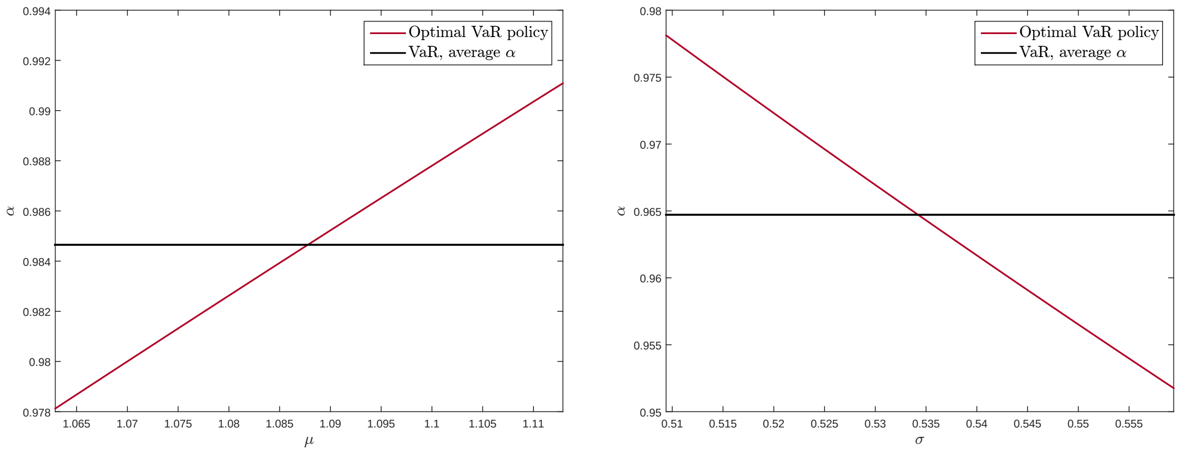
$$d^C = y - \frac{2\alpha\sigma y}{2\alpha\sigma + \beta B(\mu + (1-2\alpha)\sigma)}.$$

Therefore, if the first best is to be achieved α should be such that competitive and the first best equilibrium coincide

$$\frac{\beta\alpha y - N}{\beta\alpha + 1} = y - \frac{2\alpha\sigma y}{2\alpha\sigma + \beta B(\mu + (1-2\alpha)\sigma)}.$$

Although it is not possible to solve for confidence level α analytically, we can find how α changes with μ and σ. Figure 5a and 5b depict this relationship. Several implications of welfare analysis are noteworthy for designing optimal prudential policy. First, under our parameter calibration, we find that the confidence level is overall higher when chosen by a social planner than when fixed by regulators. If regulators imposed confidence level higher than 94% ($\approx$ 96.5% and 98.5%), this would induce the gross equilibrium interest rate decline and be lower than 1 as depicted in Figure 4a. As a result, adopting the optimal policy would have deflationary effects compared to the initial VaR policy. Second, we find that confidence level is higher when expected returns are higher and volatility is lower, and vice versa. This finding implies that regulators allow for a lower probability of default in the times of high economic growth. In terms of capital requirements, regulators impose lower capital ratios on banks when fundamentals are strong and uncertainty is low(Figure 7a and 7b). In this way, regulators increase the amount of credit available to productive firms and private sector through intermediaries. When economic growth prospects remain subdued in downturns, regulators actually discourage risk-taking and borrowing by allowing higher default probability. If deposit insurance is unavailable, the optimal policy amplifies economic contractions and expansions. In other words, the optimal policy is procyclical capital requirements.

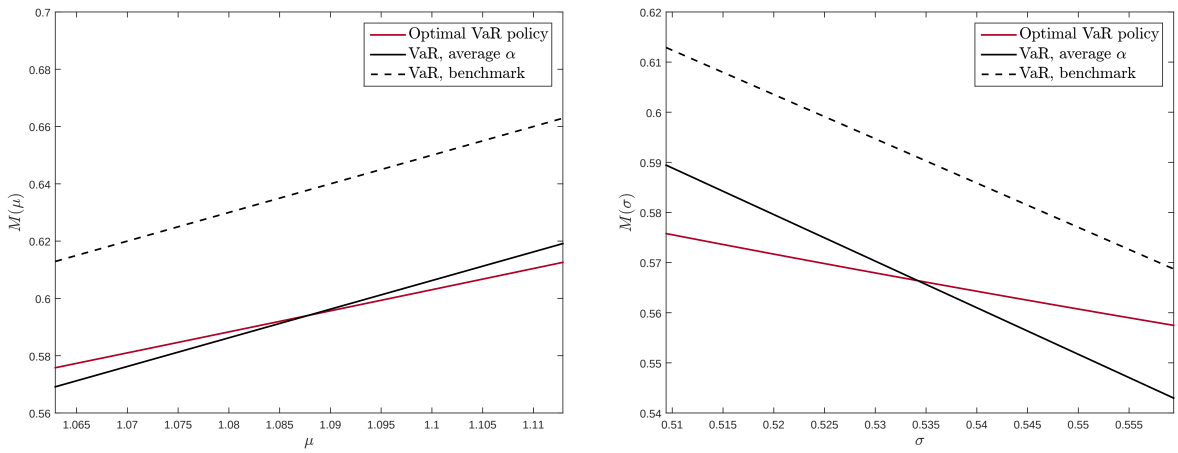
Figure 5: Optimal confidence level with VaR and uninsured deposits



(a) increase in expected return

(b) increase in volatility

Figure 6: Optimal VaR policy with uninsured deposits

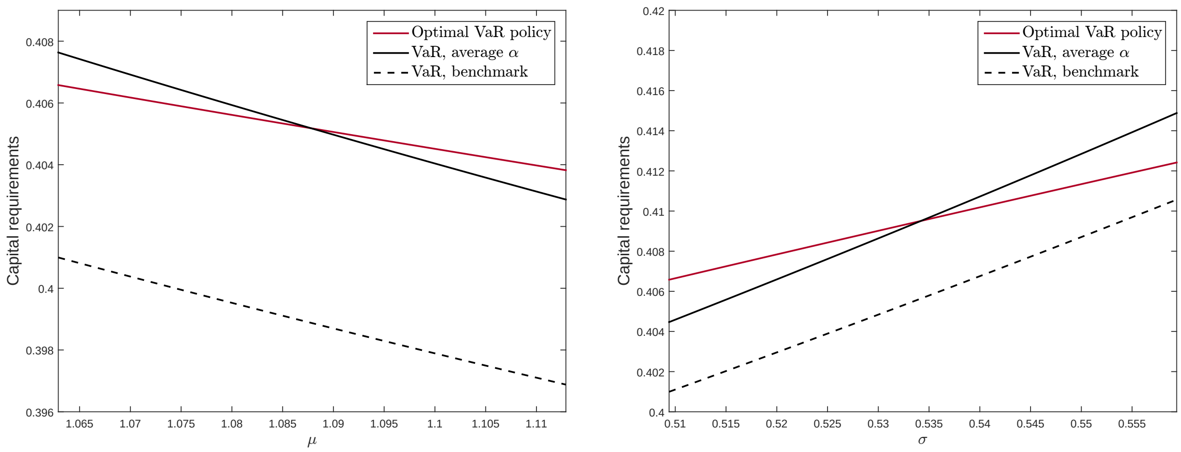


(a) increase in expected return

(b) increase in volatility

An important question is how the optimal policy differs from VaR policy with the fixed confidence level and capital regulation in the Basel III framework. When deposits are uninsured, the social planner exhibits more risk-averse behavior compared to the competitive case and capital ratios are higher. Figure 7a and 7b suggest that there is over-investment in the competitive equilibrium, and therefore, the social planner internalizes social costs of over-investment. In the context of capital regulation, capital

Figure 7: Optimal VaR capital requirements with uninsured deposits



(a) increase in expected return

(b) increase in volatility

requirements are less procyclical when the adjustment of the confidence level is possible. At impact, optimal policy resembles smoothing of the credit cycle in comparison to VaR policy with the fixed confidence level. Moreover, because market risk measures are weighted sum of moments of probability distribution, optimal policy is such that the corresponding weight to the mean is lower, while we increase the weight given to volatility[8]. As two moments proxy for the extent to which regulators can or cannot predict market outcomes, in the social planner's economy unpredictability takes precedence over predictability.

By comparison, in the Basel III framework increasing μ increases optimal capital requirements. Countercyclical capital buffers in the Basel III are justified by a prudential authority goal of limiting procyclicality of the financial sector (Freixas et al. (2015)). In this view, the financial system procyclicality(credit and prices) is seen as the main factor that contributes to the fragility of the financial system and systemic risk. In effect, our optimal policy and the Basel III accord yield opposite predictions regarding capital regulation. One possible explanation for this disagreement is the assumption that regulators still implement an optimal policy with VaR when deposits are uninsured. Although from a welfare perspective instead of systemic risk respective, we suggest that our model provides insights on how to parsimoniously implement optimal prudential policy by utilizing the existent market risk measure in Basel II and III frameworks in economies where deposit insurance is absent. Of course, this recom-

[8] This observation is straightforward if we recall the definition of VaR with the fixed confidence level $VaR_\alpha = \mu + (1 - 2\alpha)\sigma$, and VaR with the varying confidence level $VaR_{\alpha(\mu)} = \mu + (1 - 2\alpha(\mu))\sigma$ and $VaR_{\alpha(\sigma)} = \mu + (1 - 2\alpha(\sigma))\sigma$, which are weighted averages of the first and the second moment of return distribution

mendation depends on the assumption that neglecting or including market tail risk has no additional externalities apart from default costs. Implicitly, we assume there are no externalities or policies such as bailout expectations and government interventions, which we relax in the next section. Moreover, neglecting market tail risk can lead to systemic risk being underestimated in booms(Borio et al. (2001)). We leave this extension to future work.

As a final remark, we can easily prove the prediction of countercyclical default probabilities as an optimal policy. For this purpose, let us define by H the difference between the deposits chosen by the social planner and deposits from the competitive equilibrium, which is equal to zero when two equilibria coincide

$$H \equiv \frac{\beta\alpha y - N}{\beta\alpha + 1} - y + \frac{2\alpha\sigma y}{2\alpha\sigma + \beta B(\mu + (1-2\alpha)\sigma)}.$$

Using the implicit function theorem we have

$$\frac{d\alpha}{d\mu} = -\frac{\frac{\partial H}{\partial \mu}}{\frac{\partial H}{\partial \alpha}} \quad \text{and} \quad \frac{d\alpha}{d\sigma} = -\frac{\frac{\partial H}{\partial \sigma}}{\frac{\partial H}{\partial \alpha}}$$

where

$$\frac{\partial H}{\partial \alpha} = \beta\frac{N+y}{(\beta\alpha+1)^2} + \frac{2\beta B((\mu+\sigma)B - 2\alpha\sigma)y}{(2\alpha\sigma + \beta B(\mu+(1-2\alpha)\sigma))^2} > 0$$

$$\frac{\partial H}{\partial \mu} = \frac{2\alpha\beta\sigma(2\alpha\sigma - (\mu+\sigma)B)y}{(\mu+\sigma)(2\alpha\sigma + \beta B(\mu+(1-2\alpha)\sigma))^2} < 0$$

$$\frac{\partial H}{\partial \sigma} = -\frac{2\alpha\beta\mu(2\alpha\sigma - (\mu+\sigma)B)y}{(\mu+\sigma)(2\alpha\sigma + \beta B(\mu+(1-2\alpha)\sigma))^2} > 0.$$

Altogether imply that $\frac{d\alpha}{d\mu} > 0$ and $\frac{d\alpha}{d\sigma} < 0$. Intuitively, the social planner would respond more fiercely in uncertain environments compared to the competitive case where regulators focus on the fixed probability of the worst-case market outcome. In contrast, changes in fundamentals produce muted policy response.

4.3 Optimal policy with insured deposits

In this subsection, we solve for optimal risk measure if the government (monetary or prudential authority) act as a deposit insurance provider. In comparison to the previous subsection, regulators implements the optimal policy which is different from VaR. We denote a risk measure by M and assume that the probability of default is still equal to $1-\alpha$. According to this deposit regime, fixed insurance is imposed on households which pays out only in the case of bank default in the second period. In this way, deposits are fully insured. More generally, transfers in case of default proxies for unconventional monetary policies such as equity injection. Absent equity injection bankers absorb losses from a decline in fundamentals or an increase in uncertainty. With equity injection, however, losses are shared with households. On the one hand, prudential authority is concerned with risk management. On the other hand, the monetary authority provides deposit insurance when defaults occur, while insurance is financed using a fixed fee. However, the exact amount of insurance depends on how regulators measure risk. With this assumption, we implicitly suppose that there are no conflicting objectives between the prudential and the monetary authorities. Alternatively, we can think of a single prudential authority acting as a deposit insurance provider and need not distinguish between two authorities.

Related to our goal of finding the optimal prudential policy, Collard et al. (2017) derive the jointly optimal monetary and prudential policies, setting the interest rate and capital requirements. However, in our model, the central bank has the role of an insurance provider instead of enforcing the interest rate policy rule. Related to unconventional monetary policies, Gertler and Kiyotaki (2010) evaluate the effectiveness of government ex-ante equity injections in alleviating financial distress. In particular, they assume that the central bank injects equity into banks ex-ante before the crisis has occurred. In our case, we may interpret deposit insurance as ex-post equity injections, which present a complementary policy to optimal prudential policy. Since these equity injections or insurance is paid by households and equals the market risk measure, the insurance approach we design can be thought of as commitment mechanism for banks and households to raise equity on its own.

We, therefore, solve two optimization problems. First, the social planner chooses the risk measure. Second, the households and bankers optimization problem is modified when regulators impose different risk measure and deposits are insured. The social planner solves the following problem

$$\max_{M} \quad \frac{(y-d-dM)^{1-\gamma}}{1-\gamma} + \frac{\beta}{2\sigma}\int_{\mu+(1-2\alpha)\sigma}^{\mu+\sigma} \frac{\left(R^k(N+d)\right)^{1-\gamma}}{1-\gamma} dR^k + \frac{\beta}{2\sigma}\int_{\mu-\sigma}^{\mu+(1-2\alpha)\sigma} \frac{(dM)^{1-\gamma}}{1-\gamma} dR^k. \tag{29}$$

The first term represents households utility in the first period after paying the insurance fee. The second term is the expected utility in case the bank defaults, and the third term is the expected utility of equity injection when defaults occur. Since compensation equals insurance fee, the return on a unit of government equity is lower than the return on private(bank) equity. In reality, to acquire equity the central bank may pay a higher price, which is above the price of private equity, and therefore government equity earns a lower return [9]. The optimization problem can be simplified to

$$\max_{M} \quad \frac{(y-d-dM)^{1-\gamma}}{1-\gamma} + \frac{\beta}{2\sigma}B\frac{(N+d)^{1-\gamma}}{1-\gamma} + \beta(1-\alpha)\frac{(dM)^{1-\gamma}}{1-\gamma},$$

where $B = \frac{(\mu+\sigma)^{2-\gamma}}{2-\gamma} - \frac{(\mu+(1-2\alpha)\sigma)^{2-\gamma}}{2-\gamma}$. The first order condition reads

$$[M]: \quad -d(y-d-dM)^{-\gamma} + \beta(1-a)d(dM)^{-\gamma} = 0.$$

We further solve households and bankers problem. In the first period, an insurance fee T is levied on households and returned in the second period if a default occurs. Therefore, the household solves the following maximization problem

$$\max_{d} \quad \frac{(y-d-T)^{1-\gamma}}{1-\gamma} + \frac{\beta}{2\sigma}\int_{\mu+(1-2\alpha)\sigma}^{\mu+\sigma} \frac{(R^d d+\pi)^{1-\gamma}}{1-\gamma} dR^k + \frac{\beta}{2\sigma}\int_{\mu-\sigma}^{\mu+(1-2\alpha)\sigma} \frac{T^{1-\gamma}}{1-\gamma} dR^k.$$

Since households cannot choose deposit insurance fee, the last term does not affect supply of deposits. The first order condition gives

$$[d]: \quad -(y-d-T)^{-\gamma} + \frac{\beta}{2\sigma}R^d \int_{\mu+(1-2\alpha)\sigma}^{\mu+\sigma} (R^d d+\pi)^{-\gamma} = 0.$$

In equilibrium, equity injections are financed by insurance fees and budget constraint of the central bank holds, and thus $T = dM$. As before, the resource constraint implies $R^d d + \pi = R^k s$. Therefore, deposit supply is equal to

$$d = \frac{\frac{\beta}{2\sigma}BR^d y - N}{1 + \frac{\beta}{2\sigma}BR^d(1+M)}$$

Because banks still adhere to prudential regulation, deposit demand is now constrained by the new risk measure M instead of VaR_α

$$d^b = N\frac{M}{R^d - M}.$$

[9] A possible reason why the monetary authority pays a premium is that the current market price is below its average value due to financial distress (Gertler and Kiyotaki (2010)).

From the market clearing condition for deposits we obtain the equilibrium interest rate

$$R^d = \frac{\frac{\beta}{2\sigma} BM(y+N) + \frac{\beta}{2\sigma} BM^2 N + N}{\frac{\beta}{2\sigma} By},$$

and the equilibrium level of credit

$$d = \frac{\frac{\beta}{2\sigma} BMy}{1 + \frac{\beta}{2\sigma} BM(1+M)}.$$

Substituting the equilibrium amount of deposits into the social planner's first order condition and solving for M, we obtain

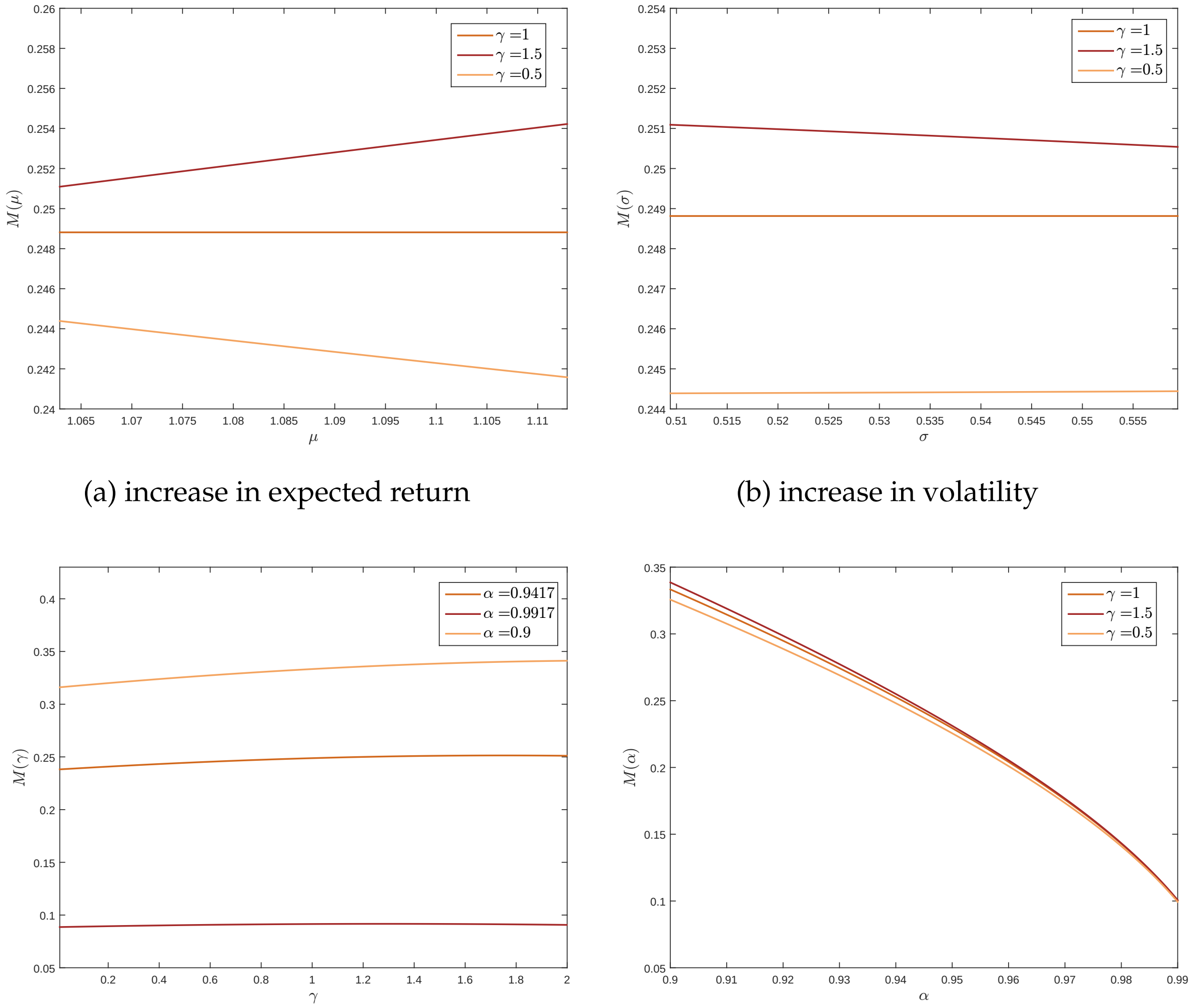
$$M = \left(\frac{2\sigma(1-\alpha)}{B} \right)^{\frac{1}{2}}$$

$$= \left(\frac{1}{1-\alpha} \int_0^{1-\alpha} (VaR_{1-p})^{1-\gamma} dp \right)^{-\frac{1}{2}}.$$

Our extension of the model with deposit insurance, while simplified, provides several useful insights. For example, we find that the optimal risk measure is inversely proportional to the average expected *utility* of tail market outcomes beyond the VaR threshold. For coherence and intuitive appeal, we call this risk measure inverse-Expected Utility Shortfall(i-EUS). The inverse proportionality may arise because with deposit insurance market losses are reflected into market gains in case of bank failure. Importantly, the social planner does not neglect market tail outcomes when deposit insurance is available.

To further understand the optimal policy, we discuss the effects of changing parameters that play an important role when designing capital requirements. The prevailing rationale for adopting countercyclical capital requirements in the Basel III regulation is the observation that default risk rises during recessions. In response to increased failure risk, risk-weighted countercyclical capital requirements raise automatically unless regulators lower capital ratios (Angeloni and Faia (2013)). We find support for the former, the optimal risk measure is decreasing in confidence level α(Figure 8d), or put differently capital requirements are increasing in default probability $1-\alpha$ (Figure 9c). In contrast to our prediction, Collard et al. (2017) characterize joint optimal prudential and monetary policy and suggest it is optimal for regulators to cut capital requirements when default risk is high because banks are naturally less tempted to

Figure 8: Optimal market risk measure with insured deposits



(a) increase in expected return

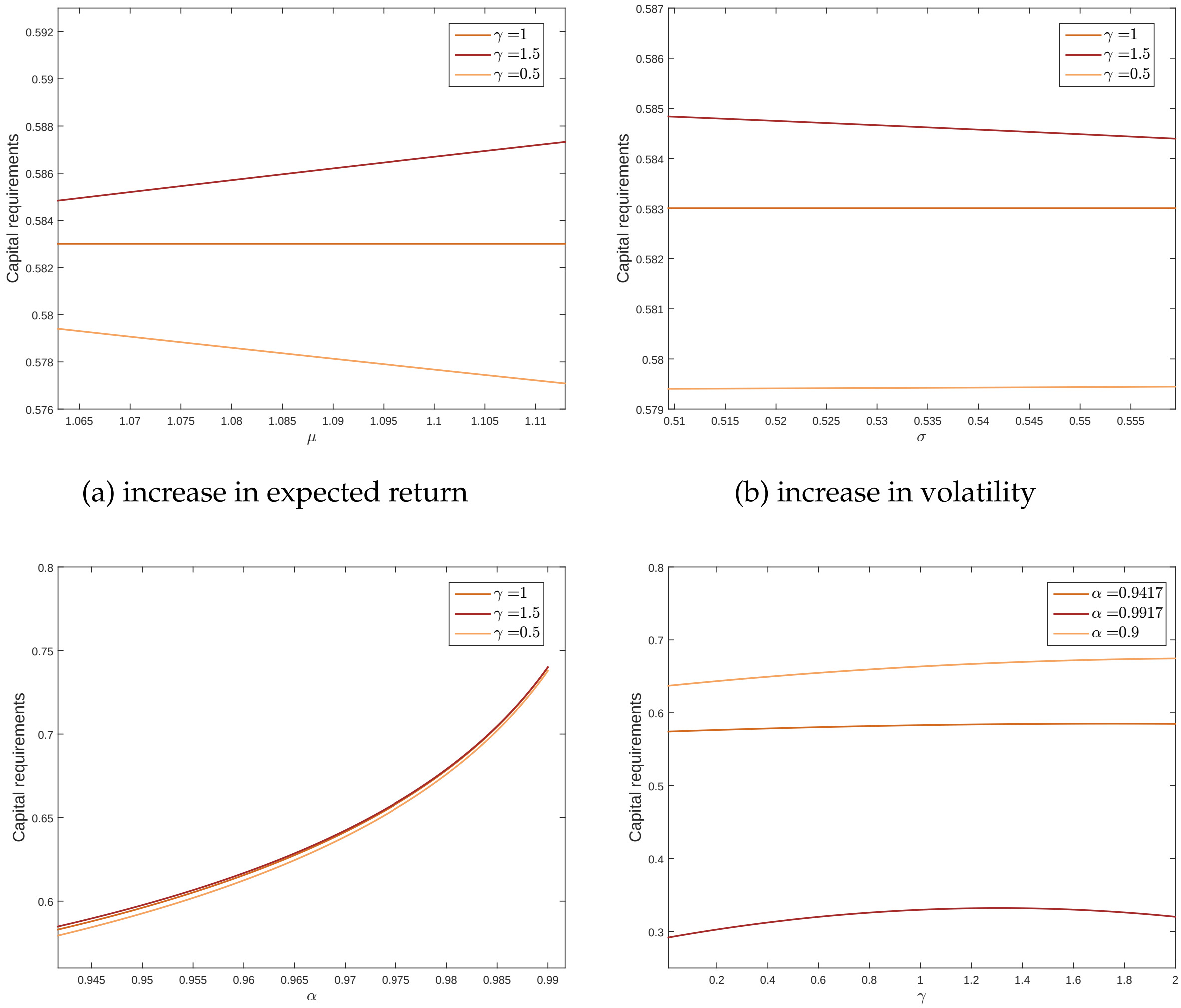
(b) increase in volatility

(c) increase in risk aversion

(d) increase in confidence level

take a risk, while optimal monetary policy raises the interest rate to dampen the expansionary effects of prudential policy. We find this thought experiment useful in the context of normative analysis of deposit insurance premiums. As argued by Acharya et al. (2010) the deposit insurance premium that exactly covers the expected cost to the deposit insurance provider, should respond to changes in individual bank default risk and systemic risk. From a normative standpoint, Acharya et al. (2010) find that the deposit insurance premium charged to banks is increasing in both risks. Although deposit premiums are paid by households in our model, this prediction means that the optimal risk measure which acts as insurance should increase with default probability. In our model, deposit insurance premiums exactly cover expected losses while these losses depend on default probability. When a commercial bank with insured deposits

Figure 9: Optimal capital requirements with insured deposits



(a) increase in expected return

(b) increase in volatility

(c) increase in non-default probability

(d) increase in risk aversion

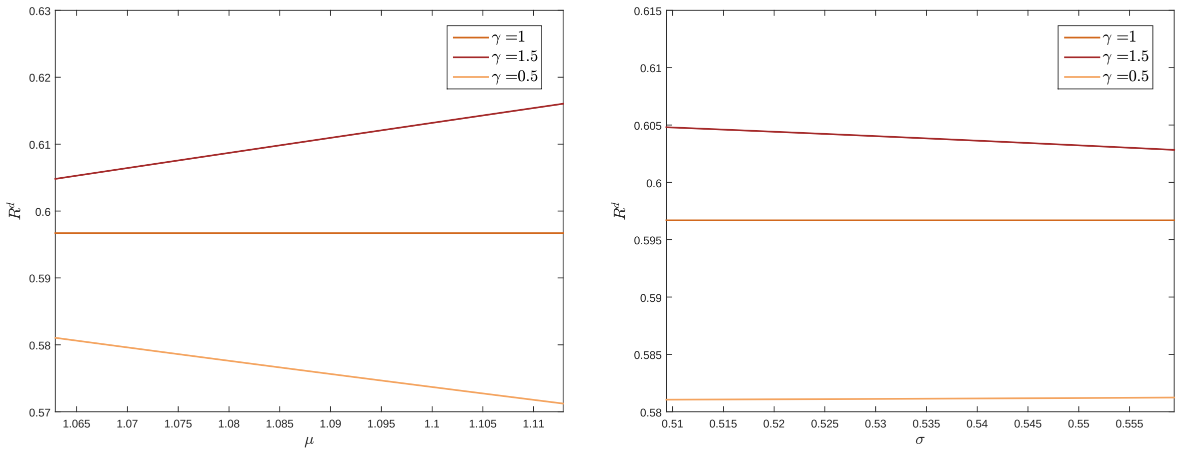
fails, the central banks take over the bank and sells bank assets or buys bank equity. In either case, during periods of widespread bank failure, the deposit insurance provider suffers from low recovery from the liquidation of failed assets or pays a higher price for bank equity. In addition, we may expect that the resolution of failed banks is more costly for the central banks when creditors become more risk-averse, directly in terms of the lower liquidation value of banks or higher equity price. This leads to our second result: the insurance charged to creditors is higher the more risk-averse they are(Figure 8c). Higher the risk aversion, higher are the corresponding capital requirements(Figure 9d).

Some main normative implications of the model without deposit insurance, qualitatively speaking, remain robust to the introduction of deposit insurance. For example, optimal risk measure is decreasing in uncertainty regardless of whether deposit insurance is available or not(Figure 8b and 6b). In terms of capital requirements, however, predictions diverge. In response to increased uncertainty, regulators cut capital requirements when deposits are insured. During times of heightened uncertainty as we have evidenced in the onset of the 2008 market downturn, it may be optimal to decrease capital requirements because heightened uncertainty may impede equity raising. Figure 8a demonstrates that an increase in the expected return can either increase or decrease optimal risk measure under deposit insurance. As a result, regulators can either cut or raise capital requirements in response to a shock that raises return on loans. A positive shock to μ increases the return on a risky asset in case banks succeed and boosts banks incentives to engage in more risk-taking. The optimal prudential policies can move in opposite directions, depending on whether creditors' risk aversion is higher or lower than one. In the first case, the optimal prudential policy is countercyclical, while in the second case, it is procyclical. A possible explanation for the ambiguity of prudential response is that bankers internalize creditors' excessive risk aversion, which diminishes banks incentives to take more risk. Overall, with deposit insurance and sufficiently risk-averse creditors, optimal capital requirements rise in response to shocks that increase bank's risk-taking incentives triggered by an increase in expected returns. We find this prediction consistent with Collard et al. (2017) and Repullo and Suarez (2012), and robust to the introduction of monetary policy or voluntary equity issuance, respectively.

In comparison to the regime when deposit insurance is absent in Figure 6a and 6b, optimal capital requirements are higher and less cyclical. Intuitively, this result may reflect a natural regulatory reaction to impose higher capital standards and buffer the economy from the financial sector failure when deposits insurance is available. The buffering is higher in the steady state, albeit lower over the cycle. Another important implication of the optimal prudential policy is that the interest rate responds in the same direction as capital requirements(Figure 10b and 10a). Among policymakers, there is a prevailing consensus on monetary and prudential policies co-movement, since both policies will be countercyclical most of the time. Collard et al. (2017) provide insights when monetary and prudential policy move in the same or opposite direction over the business cycle. In a nutshell, two policies move in the same direction when safe asset return and risky asset return are correlated, and a risk-taking channel of monetary policy is introduced. We, therefore, interpret our result consistent with positive interaction between two policies.[10]

[10]In Collard et al. (2017), prudential policy affects credit volume and composition, and bankers choose holdings of the safe and risky asset. When monetary policy affects the volume but not the composition of credit, two policies move in opposite directions. More specifically, prudential policy raises capital re-

Figure 10: The interest rate with insured deposits



(a) increase in expected return

(b) increase in volatility

To summarize our main results, our model highlights the policy instrument we think deserves more attention than presented in the existing literature: capital requirements in the form of market risk measures can directly manage risk-taking incentives. When deposit insurance is unavailable, optimal policy resembles less procyclical capital buffers by adjusting the confidence level of VaR. Optimality of procyclical buffers does not carry over when deposit insurance is available, provided that creditors are sufficiently risk-averse. At the same time, when prudential or monetary authority act as a deposit insurance provider, capital regulation mandates imposing higher capital ratios both in good and bad times.

4.4 Rationalization of Expected Shortfall

In this subsection, we provide rationale and conditions under which Expected Shortfall may be considered an optimal prudential policy. The idea is that we modify the social planner's problem and offer a possible rationalization of ES embedded in the

quirements in response to shocks that increase bank risk-taking incentives, while monetary policy cuts the interest rate in order to dampen the contractionary effects of prudential policy on credit availability to firms. The interaction between two policies then boils down to cutting interest rates to moderate the contractions caused by a rise in capital requirements. In other words, the prudential policy is contractionary and monetary policy is expansionary when shocks increase risk-taking incentives. Or put differently, the optimal prudential policy is procyclical, while optimal monetary policy is countercyclical. In contrast, when a safe asset return and risky asset return are correlated and monetary policy affects credit composition, both optimal policies are countercyclical. In this case, the interest rate responds in the same direction as capital requirements. In particular, optimal prudential policy raises capital requirements to limit risk-taking, and optimal monetary policy raises the deposit rate to dampen the effects of the investment.

Basel III accord. In the literature of risk measures, Expected Shortfall has been proposed as an alternative to overcome shortcomings of VaR, which in general is not a coherent risk measure(Acerbi and Tasche (2002)). More specifically, VaR fails to satisfy sub-additivity and monotonicity.[11] Invalidating the first property implies that portfolio risk can be higher than the sum of individual risks of assets. As a result, VaR may fail to deliver diversification benefits in an ever-growing global financial market. Moreover, VaR disregards the magnitude of tail losses once an unfavorable event occurs. One reason why regulators adopted ES in the 2008 crisis aftermath is that it includes tail market risk while VaR neglects it. Disregarding tail risk, especially in the housing market, may have systemic risk implications. In an effort to control for systemic risk, such a stringent prudential policy might have been adopted as a means to signal regulators' commitment to preventing financial crises. The question we aim to answer here is if there is a rationale for ES from a welfare perspective instead of a systemic risk perspective. By definition, ES is an average market outcome (gain or loss) exceeding the VaR limit

$$ES_{1-\alpha} = \frac{1}{1-\alpha}\int_{\mu-\sigma}^{VaR_{1-\alpha}} R^k \frac{1}{2\sigma} dR^k = \frac{1}{1-\alpha}\int_{\mu-\sigma}^{\mu+(1-2\alpha)\sigma} R^k \frac{1}{2\sigma} dR^k.$$

This integration can be done simply and we obtain

$$ES_{1-\alpha} = \frac{1}{1-\alpha}((1-\alpha)\mu + \alpha(\alpha-1)\sigma) = \mu - \alpha\sigma.$$

Notice that the difference between $VaR_{1-\alpha}$ and $ES_{1-\alpha}$ is in the weighting function, the relative weight that ES gives to volatility is $-\alpha$ and the corresponding VaR weight is $1-2\alpha$. One possible way to derive ES as an optimal market risk measure is if households are risk-neutral and the social planner solves the following maximization problem

$$\max_d \quad (y-d-T) + \frac{\beta}{2\sigma}\int_{\mu+(1-2\alpha)\sigma}^{\mu+\sigma} R^k(N+d)dR^k + \frac{\beta}{2\sigma}\int_{\mu-\sigma}^{\mu+(1-2\alpha)\sigma} dM dR^k,$$

where as before in equilibrium $T = dM$ is a fixed insurance fee imposed on households in the first period. In comparison to the case when deposits are fully insured, the social planner chooses the level of deposits instead of a risk measure. The maximization problem can be simplified to

$$\max_d \quad (y-d-T) + \beta\alpha(\mu + (1-\alpha)\sigma)(N+d) + \beta(1-\alpha)Md.$$

[11] See Definition A.1 in A.

The first order condition gives

$$[d]: \quad -1 + \beta\alpha(\mu + (1-\alpha)\sigma) + \beta(1-\alpha)M = 0.$$

The equilibrium level of borrowing is pinned down by the banker's problem, and therefore

$$d^b = N\frac{M}{R^d - M}.$$

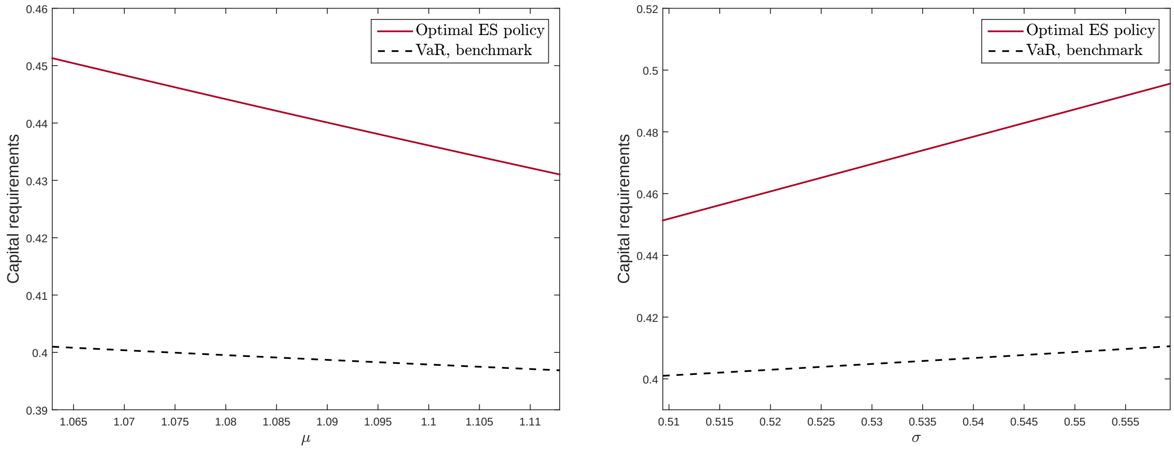
Since household are risk neutral, $R^d = \mu$. In the special case when the discount factor is equal to the inverse of the expected return on capital, i.e. $\beta = \frac{1}{\mu}$, we obtain from the first order condition

$$M = \mu - \alpha\sigma \equiv ES_{1-\alpha}.$$

If regulators adopt ES as a market risk measure, associated capital requirements would be higher and more procyclical than VaR we considered in the competitive case(Figure 11a and 11b), albeit lower than optimal capital requirements in the previous subsection. We can summarize conditions under which regulators can rationalize ES as an optimal policy. Adopting a tighter and more cyclical policy stance may result from conjunctions of three factors: creditors are risk neutral, and their time preference rate equals the expected market return. Importantly, deposit insurance is a mixture of fixed fee and variable compensation. Risk neutrality of creditors implies that they do not demand risk adjustments through the interest rate. Moreover, the second condition implies that the interest rate is equal to the inverse discount factor, and therefore, bank debt is risk-free from the creditors' perspective. Provided the latter holds, ES is optimal when households pay a fixed insurance fee to insure deposits, but in case the bank defaults compensation is paid per unit of invested deposits by the central bank or regulators. In this way, the first-period marginal utility is unaffected by deposit insurance, while it distorts the second-period marginal utility. Intuitively, ES quantifies market risk such that creditors are indifferent whether the bank defaults or not. By comparison, in the previous subsection, households treat the insurance fee and compensation as fixed. As a result, the associated deposit insurance scheme leaves households' marginal rate of substitution undistorted. In effect, i-EUS risk measure quantifies market risk such that the marginal utility of capital requirements is zero. Capital or deposit insurance is beneficial in the second period only if bank defaults, but costly in the first period as it reduces consumption.

Comparing different deposit insurance regimes, we find that it is optimal for regulators not to neglect tail market outcomes when deposit insurance is available. As a consequence, paying insurance on rare events or tail market outcomes reduces a bank

Figure 11: Optimal ES capital requirements with insured deposits



(a) increase in expected return

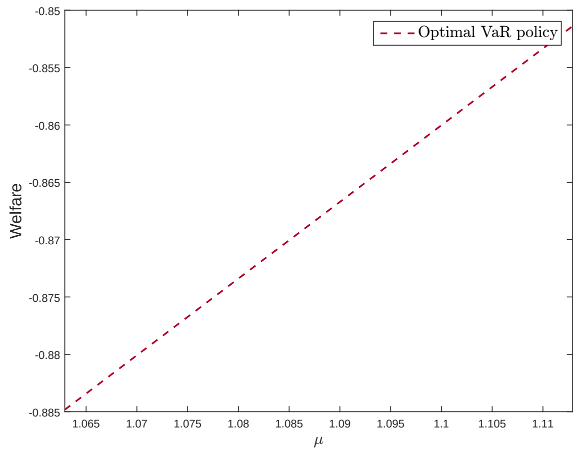
(b) increase in volatility

incentive to engage in risk-taking. Opposite to our priors, analysis indicates there may be welfare losses due to deposit insurance, which are mainly driven by the social cost of lower credit availability(Figure 12c and 12a). Consequently, making sure private sector internalizes potential adverse consequences of crises by acquiring deposit insurance seems to be suboptimal. This concern is clearly an important one, to the extent that providing the bank with liquidity in financial crises may not be a welfare-improving response. One possible reason for this result might be that bank failures generate no social cost in our model, while substantial social costs would reduce welfare when deposit insurance is unavailable. A simple way to illustrate this logic is that without capital injection from insurance paid out, individual bank failure can prompt risk related to counterparty failure. Because of bank interconnectedness through mutual debt obligations, failure of a too-big-to-fail or highly interconnected bank can destabilize the rest of the financial system. If an insolvent bank is re-capitalized instead, this could spare other banks in the borrower-lender chain. In other words, social costs such as systemic risk or bank failure resolution are absent in our welfare analysis. Without these social costs, the level of calculated welfare entails positive bias under VaR regulation.[12] Another explanation is that we set the confidence level to the one from the benchmark model calibration when we evaluate the welfare impact of the alternative policy reform, which may not be an appropriate choice in the new economy with deposit insurance and a different market risk measure. The key to a welfare-improving

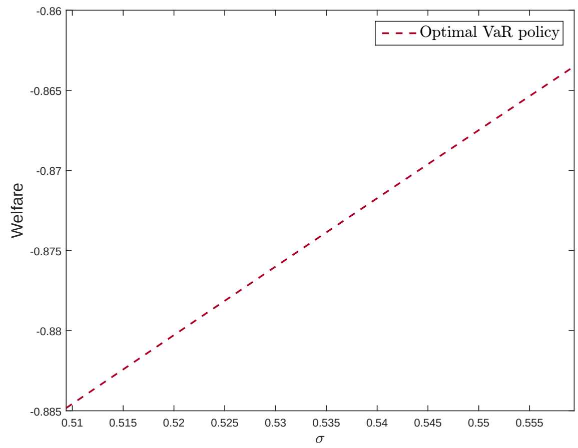
[12]One possible estimate for these social costs could be the loss given default(LDG) parameter from Basel II Internal Ratings-Based (IRB) which we have used in the calibration of parameters, and equal to 0.45. In that case, introducing deposit insurance and an alternative market risk measure implies a welfare-improving prudential response.

aspect of VaR policy is that ignoring tail market risk is beneficial when there is no authority to act as an insurance provider. Overall, our result suggests that providing deposit insurance is suboptimal in the absence of significant social costs of bank default. In order for deposit insurance to be beneficial, creditors need to believe that they could be forced to absorb significant losses if a bank is close to insolvency. For instance, this can be achieved if creditor claims are converted to equity claims. The commitment of invoking this resolution scheme, called “bail-in,” is planned by regulators to be implemented through post-crisis legislation in Title II of the US Dodd-Frank Act(Duffie (2019)).

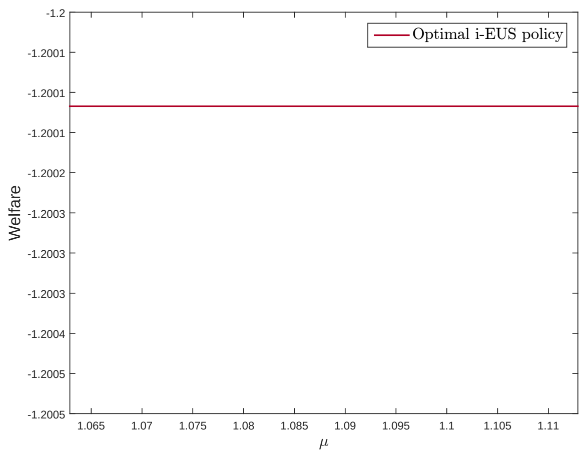
Figure 12: Welfare with optimal VaR and i-EUS capital requirements



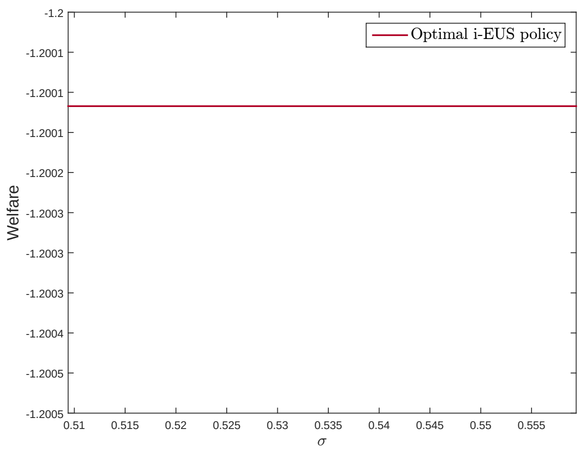
(a) increase in expected return



(b) increase in volatility



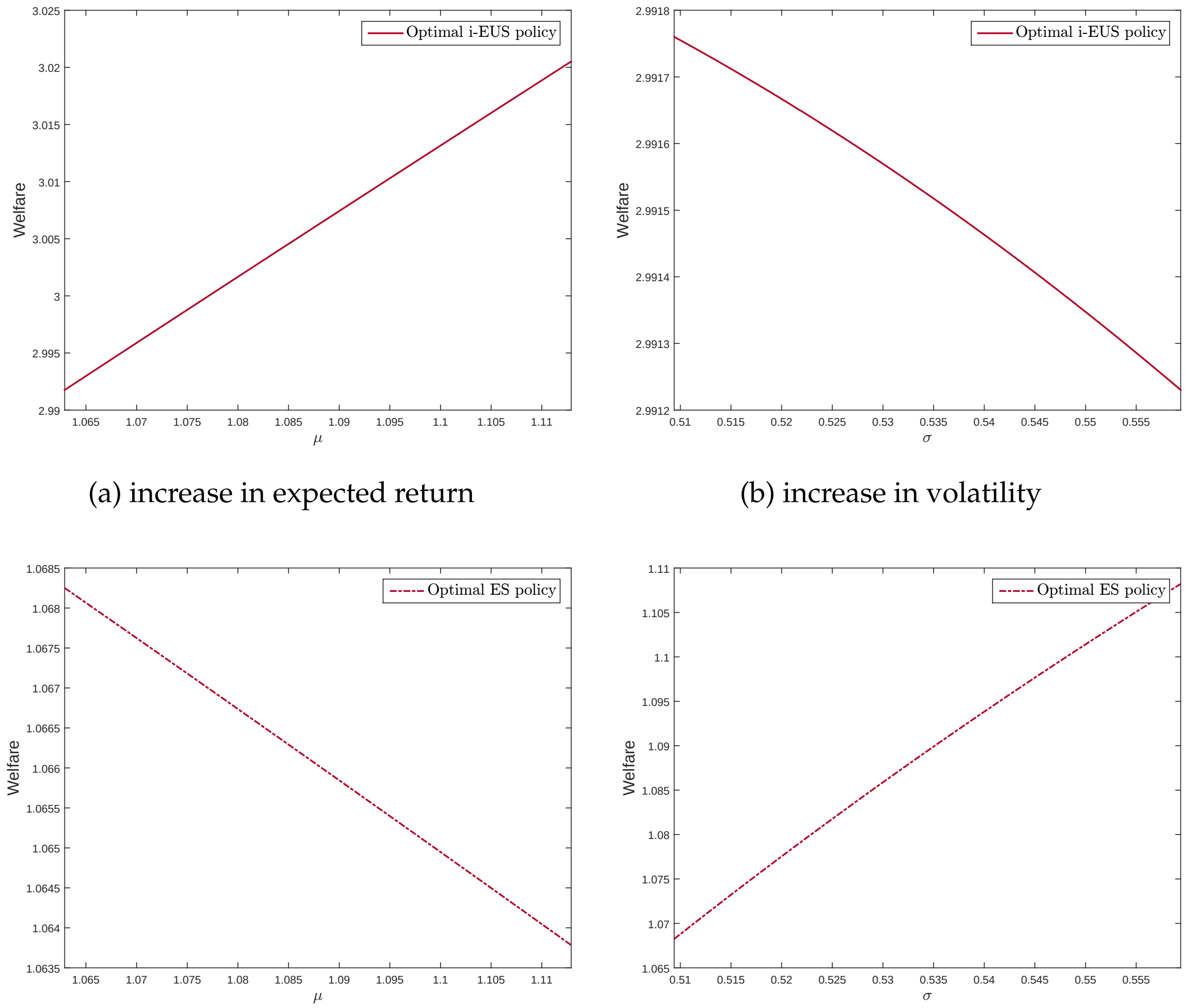
(c) increase in expected return



(d) increase in volatility

This figure depicts welfare for different values of the expected return and volatility. For optimal VaR and i-EUS policy, risk aversion is set to 1. The confidence level for optimal i-EUS policy is the same as in Table 1, whereas for optimal VaR policy the confidence level changes as in Figure 5a and 5b.

Figure 13: Welfare with optimal ES and i-EUS capital requirements



(a) increase in expected return

(b) increase in volatility

(c) increase in expected return

(d) increase in volatility

This figure depicts welfare for different values of the expected return and volatility. For the optimal ES policy households are risk neutral, while for the optimal i-EUS policy risk aversion is set to $\frac{1}{2}$. The confidence level for optimal i-EUS and ES policy is the same as in Table 1.

If however, the mission of regulators is to build a financial system where institutions can absorb adverse shocks with minimum negative spillovers to welfare, or extract welfare gains from improved growth prospects, an adequate choice of a prudential tool and insurance regime is less than a straightforward answer. Figure 12c and 12d suggest that for households with log preferences and fixed deposit insurance, welfare is invariant to changes in the market environment. By comparison, with no deposit insurance and VaR risk measure, welfare rises in the face of higher uncertainty(Figure 12b). Moreover, with VaR policy economy as a whole has the ability to absorb market gains(Figure 12a). The main reason for this aspect of VaR policy is that for households with log preferences it is socially optimal that deposit choice is independent of market return or volatility, while the expected utility of per unit of risky security increases with both factors. For myopic creditors with log preferences, regulators have a choice between welfare resilience(i-EUS) versus welfare amplification(VaR). At the core of this result lies identification of regulatory regime under which prudential authority and lenders are myopic. The tendency to underestimate or ignore the probability of low-frequency events such as financial crises is known as disaster myopia. The idea is that, by quantifying market risk with VaR, susceptibility to disaster myopia is reinforced by prudential institutional factors since VaR neglects tail market risk. When disaster myopia sets in, lenders ignore bank insolvency risk even though they do not believe they will be protected in a case a disaster such as a bank failure occurs. With such perceptions, lenders believe they could accept weaker government guarantees without increasing market losses. As market volatility increases, due to disaster myopia lenders ignore the likelihood of higher market losses. Lenders act myopically as if investment opportunities are constant with log preferences and focus on the possibility of capturing higher market gains when volatility rises. As a result, when creditors and regulators are both myopic, an increase in volatility leads to welfare enhancement(Figure 12b). When lenders act myopically but do not exhibit behavior in congruence with disaster myopia, lenders receive appropriate provision for potential market losses and welfare is independent of variations in market return and volatility(Figure 12c and 12d).

A belief by creditors that banks are no longer too-big-to-fail leads to the inferior alignment of the risk-taking incentives with social incentives. As disaster myopia disappears and risk aversion of creditors diminishes, regulators need to discern between priorities of market(bank and creditors) discipline and negative welfare spillovers. We find that adopting ES in the Basel III is justified as long as welfare spillovers from market instability are the major concerns for regulators. If, however, regulators aim to deter financial sector and creditors from taking riskier investments, forcing creditors to buy fixed insurance may be an adequate choice. Given the imperfections of regulation to account for both priorities, we argue that a useful complement to purely regulatory risk management is for the government to adopt a discretionary approach when implementing policies. By relying on these two priorities, regulators may find it valuable to switch between fixed and variable insurance regime over the financial cycle. In particular, to prevent banks from taking excessively risky projects in ”reach for yield” on the asset side of their balance sheets, regulators may implement fixed insurance before the financial crisis occurs. Once the crisis materializes, which inevitably leads to higher uncertainty and depressed market returns, regulators should switch to variable insurance regime. In this way, in the pre-crisis years, high-risk investments are penalized while market gains are absorbed (Figure 13b and 13a); whereas during the crisis not only negative spillovers of heightened market instability are dampened, but positive welfare spillovers are generated. To elaborate more on this point, with optimal i-EUS policy creditors are indifferent whether or not to buy insurance against bank insolvency, while with ES they are indifferent whether bank fails or not in the second period. The choice of prudential tool should follow predictable intermediate-term momentum and long-term reversal in stock prices. The periods of strong market growth are the precise time when insurance against future crashes should be bought, and quantifying market risk with i-EUS achieves this objective. Once price-reversal becomes evident, regulators ought to switch to ES in risk management practices. In market downturns, welfare actually rises when creditors are indifferent to the possibility of future bank failure(Figure 13c). This rise occurs because households consumption in the second-period decrease as market gains become depressed, but creditors discount the future consumption less and less. In other words, future consumption becomes more valuable as creditors become more patient. Of course, the proposal of alternating deposit insurance regime over the financial cycle involves a degree of regulatory discretion rather than a commitment to one market risk measure independent of the stage of the financial cycle. Implementing contingent prudential policy might be difficult if ”rules rather than discretion ” approach is emphasized by the prudential authority. Alternatively, regulators might average out across two regimes, and impose deposit insurance scheme with both fixed and variable compensation in case of bank insolvency.

Before concluding, let us convey final concern about our proposed prudential design. In our approach of capital regulation welfare perspective is emphasized while abstracting from fire-sale and credit crunch externalities. With a such narrow supervisor’s objective, higher capital requirements inevitable lead to curtailing of bank lending to private businesses. Moreover, when high market losses erode bank capital or equity, banks can adjust to capital regulation either by raising new equity or selling assets. If the individual bank chooses to liquidate the asset, it can cause its price to plummet and lead to fire sales spiral, which may trigger other banks to deleverage. If despite best efforts, regulators cannot prevent fire sales or credit crunch, they might benefit from focusing on welfare benefits. We have offered several specific proposals of optimal market risk measures, based on deposit insurance availability and creditors’ risk aversion. Accounting for the above externalities and testing for the robustness of proposed market risk measures is the important step in prudential policy design before such policies are implemented by regulators.

5 Conclusion

The optimal prudential policy in the form of market risk measure is a key issue in policy design that has received limited attention in the literature. In the first part of the paper, we considered a simple macro model with financial sector under VaR capital requirements embedded in a form of market risk measure. We have demonstrated that solely focusing on the worst-case market outcome while neglecting tail outcomes with VaR when designing prudential policy, is potentially the main driver of procyclical leverage in the banking sector. The goal of the optimal prudential policy is to maximize welfare by encouraging or discouraging risk-taking but to accomplish this objective through market risk measures and deposit insurance design. When deposits are uninsured, the welfare is maximized if capital requirements are state dependent. The optimal policy implies that regulators allow for a lower probability of default in the times of high economic growth and low uncertainty. At impact, optimal policy resembles smoothing of the credit cycle in comparison to VaR with the fixed confidence level. When monetary authority or regulators act as a deposit insurance provider, capital regulation mandates imposing higher capital ratios. Provided that creditors are sufficiently risk averse, the optimal policy is countercyclical. In contrast, if risk aversion of creditors is below one, the optimal policy is procyclical. Finally, we provide conditions for optimality of Expected Shortfall implemented in the Basel III accord. From a welfare perspective, Expected Shortfall is the optimal policy when creditors are risk neutral and deposit insurance is such that fixed fee is levied on creditors, while insurance is paid out to creditors per unit of deposits in the case of bank default. Our main message is that providing deposit insurance is suboptimal in terms of welfare levels in the absence of significant social costs of bank failure. Comparing different insurance regimes, we find that it is optimal for regulators not to neglect tail market outcomes when creditors are protected by deposit insurance.

Some extensions of the simple model are worthy of pursuit in the future. First, since capital regulation affects risk-taking channel and welfare, the optimal interaction of conventional monetary and prudential policy can be analyzed. For this purpose, developing an extension that incorporates a risk-taking channel of monetary policy could be a promising direction. Second, our model abstracts from systemic risk, a prevailing rationale for regulating the banking sector in the first place. Optimal policies from a welfare perspective may not be optimal from a systemic risk perspective. A prudential authority which incorporates implications of market risk measures both on welfare and systemic risk may be a way forward in prudential policy design.

References

Acerbi, C. (2002). Spectral measures of risk: A coherent representation of subjective risk aversion. *Journal of Banking & Finance*, 26(7):1505–1518.

Acerbi, C. and Tasche, D. (2002). On the coherence of expected shortfall. *Journal of Banking & Finance*, 26(7):1487–1503.

Acharya, V. V. (2009). A theory of systemic risk and design of prudential bank regulation. *Journal of financial stability*, 5(3):224–255.

Acharya, V. V., Santos, J., Yorulmazer, T., et al. (2010). *Systemic risk and deposit insurance premiums*. Citeseer.

Adrian, T. and Brunnermeier, M. K. (2011). Covar. Technical report, National Bureau of Economic Research.

Adrian, T. and Shin, H. S. (2010). Liquidity and leverage. *Journal of financial intermediation*, 19(3):418–437.

Angeloni, I. and Faia, E. (2013). Capital regulation and monetary policy with fragile banks. *Journal of Monetary Economics*, 60(3):311–324.

Artzner, P., Delbaen, F., Eber, J.-M., and Heath, D. (1999). Coherent measures of risk. *Mathematical finance*, 9(3):203–228.

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, 1:1341–1393.

Borio, C., Furfine, C., Lowe, P., et al. (2001). Procyclicality of the financial system and financial stability: issues and policy options. *BIS papers*, 1(March):1–57.

Brunnermeier, M. K. and Sannikov, Y. (2014). A macroeconomic model with a financial sector. *American Economic Review*, 104(2):379–421.

Collard, F., Dellas, H., Diba, B., and Loisel, O. (2017). Optimal monetary and prudential policies. *American Economic Journal: Macroeconomics*, 9(1):40–87.

Committee, B. et al. (2010). Guidance for national authorities operating the countercyclical capital buffer. *Basel, Switzerland*.

Drehmann, M., Borio, C. E., Gambacorta, L., Jimenez, G., and Trucharte, C. (2010). Countercyclical capital buffers: exploring options.

Drehmann, M. and Gambacorta, L. (2012). The effects of countercyclical capital buffers on bank lending. *Applied economics letters*, 19(7):603–608.

Duffie, D. (2019). Prone to fail: The pre-crisis financial system. *Journal of Economic Perspectives*, 33(1):81–106.

Elsby, M. W., Hobijn, B., and Sahin, A. (2010). The labor market in the great recession. Technical report, National Bureau of Economic Research.

Freixas, X., Laeven, L., and Peydró, J.-L. (2015). *Systemic risk, crises, and macroprudential* *regulation*. MIT Press.

Gennaioli, N. and Shleifer, A. (2018). *A crisis of beliefs: Investor psychology and financial fragility*. Princeton University Press.

Gertler, M. and Kiyotaki, N. (2010). Financial intermediation and credit policy in business cycle analysis. In *Handbook of monetary economics*, volume 3, pages 547–599. Elsevier.

He, Z. and Krishnamurthy, A. (2014). A macroeconomic framework for quantifying systemic risk. Technical report, National Bureau of Economic Research.

Kashyap, A., Rajan, R., Stein, J., et al. (2008). Rethinking capital regulation. *Maintaining stability in a changing financial system*, 43171.

Laeven, M. L. and Valencia, F. (2010). *Resolution of banking crises: The good, the bad, and the ugly*. Number 10-146. International Monetary Fund.

Markowitz, H. (1952). Portfolio selection. *The journal of finance*, 7(1):77–91.

Mizen, P. (2008). The credit crunch of 2007-2008: a discussion of the background, market reactions, and policy responses. *Federal Reserve Bank of St. Louis Review*, 90(September/October 2008).

Repullo, R. and Saurina Salas, J. (2011). The countercyclical capital buffer of basel iii: A critical assessment.

Repullo, R. and Suarez, J. (2012). The procyclical effects of bank capital regulation. *The Review of financial studies*, 26(2):452–490.

Tversky, A. and Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and uncertainty*, 5(4):297–323.

Wang, S. S. (2000). A class of distortion operators for pricing financial and insurance risks. *Journal of risk and insurance*, pages 15–36.

A Omitted proofs and definitions

In this appendix, we present definitions and omitted proofs used in the main text. A risk measure is defined by the following four coherency axioms (Artzner at al., 1999). Suppose that X is a random variable that represents a future uncertain profit or a loss.

Definition A.1. (Risk measure) Let V be a set of all real-valued random variables on a probability space (Ω, Σ, P). A function $\rho : V \to \mathbb{R}$ is called a coherent risk measure if it satisfies following axioms:

i) (Translation invariance) For all $X \in V$ and all $a \in \mathbb{R}$, $\rho(X + a) = \rho(X) - a$.

ii) (Subadditivity) For all X and $Y \in V$, $\rho(X + Y) \leq \rho(X) + \rho(Y)$.

iii) (Positive homogeneity) For all $h \geq 0$, $\rho(hX) = h\rho(X)$.

iv) (Monotonicity) For all X and $Y \in V$, with $X \leq Y$, we have $\rho(Y) \leq \rho(X)$.

Definition A.2. (Quantile) Lower α-quantile of a random variable X is defined by

$$q_\alpha(X) = \inf\left\{x \in \mathbb{R} : \quad P[X \leq x] \geq \alpha\right\}, \quad \alpha \in (0,1).$$

Proof of Proposition 1. Let us rewrite the market clearing condition (24) and define H as follows

$$H \equiv N\frac{F^{-1}(1-\alpha)}{R^d - F^{-1}(1-\alpha)} - \frac{\left(\frac{BR^d\beta}{2\alpha\sigma}\right)^{\frac{1}{\gamma}} y - N}{1 + \left(\frac{BR^d\beta}{2\alpha\sigma}\right)^{\frac{1}{\gamma}}} = 0.$$

For simplicity, we will prove the proposition for log-preferences. To prove that R^d is increasing in μ, we use the implicit function theorem

$$\frac{dR^d}{d\mu} = -\frac{\frac{\partial H}{\partial \mu}}{\frac{\partial H}{\partial R^d}}.$$

$$\frac{\partial H}{\partial \mu} = N\frac{F_\mu^{-1}(1-\alpha)(R^d - F^{-1}(1-\alpha)) + F_\mu^{-1}(1-\alpha)F^{-1}(1-\alpha)}{\left(R^d - F^{-1}(1-\alpha)\right)^2}$$

$$+\frac{2\alpha\beta\sigma R^d y}{(\mu+\sigma)(\mu+(1-2\alpha)\sigma)(2\alpha\sigma+\beta BR^d)} + \frac{2\alpha\beta\sigma R^d(2\alpha\sigma N - \beta BR^d y)}{(\mu+\sigma)(\mu+(1-2\alpha)\sigma)(2\alpha\sigma+\beta BR^d)^2}$$

$$= \frac{R^d N F_\mu^{-1}(1-\alpha)}{\left(R^d - F^{-1}(1-\alpha)\right)^2} + \frac{4\alpha^2\beta\sigma^2 R^d (N+y)}{(\mu+\sigma)(\mu+(1-2\alpha)\sigma)(2\alpha\sigma+\beta B R^d)^2}.$$

Similarly,

$$\frac{\partial H}{\partial R^d} = -\frac{N(F^{-1}(1-\alpha))}{\left(R^d - F^{-1}(1-\alpha)\right)^2} - \frac{\beta B y}{\beta B R^d + 2\alpha\sigma} + \frac{\beta B(\beta B R^d y - 2\alpha\sigma N)}{(\beta B R^d + 2\alpha\sigma)^2}$$

$$= \frac{-N F^{-1}(1-\alpha)}{\left(R^d - F^{-1}(1-\alpha)\right)^2} - \frac{2\alpha\sigma\beta B(N+y)}{(\beta B R^d + 2\alpha\sigma)^2}.$$

In order to prove that $\frac{dR^d}{d\mu} > 0$, we need the sign of $F_\mu^{-1}(1-\alpha)$, which denotes for the derivative with respect to μ of the inverse cumulative distribution function of uniform distribution at the point $1-\alpha$. The inverse of the cumulative distribution function is known as the quantile function, and in the case of uniform distribution it can be expressed as $F^{-1}(1-\alpha) = \mu + (1-2\alpha)\sigma$. Therefore, $F_\mu^{-1}(1-\alpha) = 1$, and altogether implies $\frac{dR^d}{d\mu} > 0$. □

Proof of Proposition 2. Leverage of the banking sector is defined by (27). Then we have

$$\frac{dL}{d\mu} = \frac{\frac{dR^d}{d\mu}(R^d - F^{-1}(1-\alpha)) - R^d\left(\frac{dR^d}{d\mu} - \frac{dF^{-1}(1-\alpha)}{d\mu}\right)}{(R^d - F^{-1}(1-\alpha))^2}$$

$$= \frac{R^d - F^{-1}(1-\alpha)\frac{dR^d}{d\mu}}{(R^d - F^{-1}(1-\alpha))^2}.$$

From the proof of Proposition 1, we can calculate $\frac{dR^d}{d\mu}$, which yields

$$\frac{dR^d}{d\mu} = \frac{\frac{4\alpha^2\sigma^2 N}{(\mu+\sigma)(\mu+(1-2\alpha)\sigma)} + \beta(N+y)B^2}{\beta y B^2}.$$

We obtain

$$R^d - F^{-1}(1-\alpha)\frac{dR^d}{d\mu} = \frac{2\alpha\sigma N + \beta B(\mu+(1-2\alpha)\sigma)(y+N)}{\beta B y} - (\mu+(1-2\alpha)\sigma)\frac{\frac{4\alpha^2\sigma^2 N}{(\mu+\sigma)(\mu+(1-2\alpha)\sigma)}}{\beta y B^2}$$

$$= \frac{2\alpha\sigma N((\mu+\sigma)B - 2\alpha\sigma)}{\beta B^2(\mu+\sigma)y} > 0,$$

which implies that leverage is an increasing function of the expected return μ. $\square$

Proof of Proposition 3. Analogously to the proof of Proposition 2, using the expression for leverage defined by (27), we have

$$\frac{dL}{d\sigma} = \frac{\frac{dR^d}{d\sigma}(R^d - F^{-1}(1-\alpha)) - R^d\left(\frac{dR^d}{d\sigma} - \frac{dF^{-1}(1-\alpha)}{d\sigma}\right)}{(R^d - F^{-1}(1-\alpha))^2}$$

$$= \frac{R^d F_\sigma^{-1}(1-\alpha) - F^{-1}(1-\alpha)\frac{dR^d}{d\sigma}}{(R^d - F^{-1}(1-\alpha))^2},$$

where $F_\sigma^{-1}(1-\alpha)$ denotes the derivative of the quantile function with respect to σ. This gives $F_\sigma^{-1}(1-\alpha) = \frac{d(\mu+(1-2\alpha)\sigma)}{d\sigma} = 1-2\alpha$. What is left to calculate is $\frac{dR^d}{d\sigma}$ which is equal to

$$\frac{dR^d}{d\sigma} = \frac{\left(\frac{-2\alpha\sigma(2\alpha\sigma + \beta(\mu+(1-2\alpha)\sigma)(N+y)B)}{(\mu+\sigma)(\mu+(1-2\alpha)\sigma)} + (2\alpha\sigma + \frac{2\alpha\beta\mu(N+y)}{\mu+\sigma} + (1-2\alpha)\sigma(N+y)B)B\right)}{\beta y B^2}$$

Finally, we obtain

$$(1-2\alpha)R^d - F^{-1}(1-\alpha)\frac{dR^d}{d\sigma} = \frac{-2\alpha\mu N((\mu+\sigma)B - 2\alpha\sigma)}{\beta B^2(\mu+\sigma)y} < 0.$$

This concludes the proof that leverage is a decreasing function of the volatility σ. $\square$